Behaviour-based price discrimination: the impact of an information asymmetry

by

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Vita

Filipa Alexandra Pereira Rodrigues was born on May 20^{th}, 1994, in Porto, Portugal.

After completing the secondary education, in 2012, at Colégio de Nossa Senhora do Rosário, in the area of Socioeconomics Sciences, with the average of 20 (out of 20), Filipa has continued her studies in Economics at Faculdade de Economia da Universidade do Porto (FEP). In 2015, the Bachelor in Economics was completed with an average of 17 (out of 20).

With the aim of deepening her knowledge on Economics, as well of acquiring the necessary abilities to pursue a solid career in the area, the Masters in Economics, once again in FEP, was the next logical step to take.

Currently a finalist of this Master's degree, Filipa is motivated to start her first professional experience, working for Deloitte Portugal, in September 2017.

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Abstract

In the past, personalized price discrimination used to be a just theoretical topic, with little possibility of actually being implemented. However, with the recent technological evolution this theoretical topic became a reality. Nowadays, companies have the ability of gathering information about their customers' behaviour, which can be used to segment them and price discriminate accordingly.

The aim of this dissertation is to analyze the impact of behaviour-based pricing on price strategies and welfare outcomes when there is an information asymmetry between firms. In order to achieve my objectives I will use two set-ups of a static duopoly model (with and without information asymmetry), inspired in Gehrig *et al.* (2011), and compare them with each other and with Gehrig *et al.* (2011). This research gives new insight on the effects of behaviour-based price discrimination and on the relevant aspects that impact firms and consumers, such as market growth and past choices.

The results show that the entry of new customers in a market with information asymmetry between firms benefits the non-discriminating firm and helps reduce the strategic advantage the other firm has. This entry also plays an important (and somewhat similar) role when no information asymmetry is present, as it attenuates the dominance of the stronger firm. Moreover, the existence of an information asymmetry always benefits the firm that has the technological advantage and, in many cases, harms the rival. For consumers, the information asymmetry can be beneficial. Lastly, incorporating past decisions into the pricing strategies is always positive for consumers, but usually harms the firms. Our model, however, finds an exception to these results. When there is a strong preference for the price discriminating firm, the other firm might be benefitted by the inclusion of this aspect on the pricing strategies.

JEL codes: D43, L11, L13

Keywords: Price discrimination, Behaviour-based price discrimination, Customer Poaching, Information asymmetry

Resumo

No passado, a discriminação de preços personalizada mais não era do que um tema teórico, com pouca possibilidade de ser aplicado na prática. Contudo, com a recente evolução tecnológica, essa possibilidade teórica tornou-se uma realidade. Atualmente, as empresas têm a capacidade de reunir informação sobre o comportamento dos consumidores, podendo segmentá-los e discriminar preços de acordo com esses segmentos.

O objetivo desta dissertação é analisar o impacto da discriminação de preços com base no comportamento dos consumidores nas estratégias de preços e no bem-estar, num contexto em que existe assimetria de informação entre as empresas. Para esse efeito, utilizarei um modelo de duopólio estático com duas variantes (com e sem assimetria de informação), inspirado no modelo de Gehrig *et al.* (2011) e compararei os três entre si. Este trabalho permitirá obter novas conclusões sobre os efeitos deste tipo de discriminação de preços e sobre os aspetos, como o crescimento dos mercados e as opções passadas, que influenciam empresas e consumidores.

Os resultados mostram que a entrada de novos consumidores num mercado com assimetria de informação ajuda a reduzir a vantagem estratégica da empresa com acesso a mais informação, beneficiando a outra empresa. Esta entrada desempenha também um papel fulcral (e, de certa forma, semelhante) quando não existe assimetria de informação, pois atenua o poder de mercado da empresa dominante. Além disso, a assimetria beneficia naturalmente a empresa capaz de discriminar preços e, em muitos casos, prejudica a rival, enquanto para os consumidores pode ser benéfica. Por último, incorporar as decisões passadas no processo de decisão estratégica dos preços é sempre positivo para os consumidores, mas tende a prejudicar as empresas. Existe, contudo, uma exceção: quando há uma preferência clara pela empresa que discrimina preços, a outra empresa pode sair beneficiada com a inclusão desse aspeto na definição das estratégias de preços.

Códigos-JEL: D43, L11, L13

Palavras-chave: Discriminação de preços, Discriminação de preços com base no comportamento dos consumidores, “Poaching” dos consumidores, Assimetria de informação

Table of contents

List of figures

1. Introduction

In the past, personalized price discrimination was merely a theoretical topic as it was very hard to implement. However, due to technological evolution, nowadays companies have the ability to collect and trade big data and to communicate directly with customers. This allows them to segment their customers based on past behaviour and price discriminate accordingly.[1] This new reality raises a number of issues regarding the optimal firms' behaviour, its impact on consumers and the possible need for authorities to intervene in markets where such practices are common. For this reason, personalized price discrimination became an increasingly studied topic not only among academics, but also between big international institutions.

According to the existent literature, in the case of a monopoly, price discrimination unequivocally reduces the consumer surplus. However, when companies face competition, the same may not remain true. For example, in the case of an oligopoly, literature predominantly defends that such practice benefits consumers, since it will lead to lower prices in order to attract customers who, otherwise, would prefer to buy the good from a rival company. As a result, political intervention in these cases is, in general, viewed as unnecessary.

With this dissertation, I intend to deepen the study of behaviour-based price discrimination. I will focus not only on situations where all firms have the same ability to price discriminate, but also on cases in which there is an information asymmetry between companies regarding the customers' past behaviour. This will allow me to understand what changes in price and welfare strategies when there is asymmetry on the firms' ability to customize prices. For this purpose, I will use Gehrig *et al* (2011) as a benchmark. Nevertheless, I will depart from that paper in one important aspect. While Gehrig *et al* (2011) study a duopoly market (with asymmetric information) where one of the firms used to always be a monopolist in the past, this dissertation studies a context where previous competition was held in the past, leaving firms with an inherited group of loyal clients. Given that consumers have selected their preferred firm in the past, in this dissertation I will take into consideration the customer's past decisions as an element that

[1] Executive Office of the President of the United States, Council of Economic Advisers (2015) mentions the new technological setting that facilitates personalized price discrimination, as well as the consequent authorities' concerns.

influences the behaviour of the companies. Therefore, the dissertation will answer two questions. First, what is the impact of asymmetry on the degree of firm's customized pricing effectiveness on market equilibrium (prices and profits) and on consumer welfare? And second, how does the consumer's past behaviour influences the strategic decisions of the firms and, consequently, their profits?

The rest of this dissertation will be divided in three parts. In section 2, I will present my literature review, including the key concepts of my dissertation, the historical context of the research questions and an analysis of the different contributions to the literature on behaviour-based price discrimination. Section 3 will be dedicated to the presentation of our model: section 3.1 presents the asymmetric behaviour-based price discrimination model and the analysis of its consequences in terms of prices, profits and consumer surplus; section 3.2 does the same for the scenario without information asymmetry; section 3.3 contains a comparison between those scenarios; and, finally, in section 3.4 there will be a comparison between our model and Gehrig *et al* (2011). Conclusions and suggestions for possible future lines of investigation are presented in section 4.

2. Literature review

Price discrimination has been a subject of study among economists for many years. A firm is price discriminating when it sets different prices for different consumers without any cost related reason. There are three main types of price discrimination: first degree price discrimination, which consists in charging the maximum price that each customer is available to pay and, therefore, capturing the entire surplus; second degree price discrimination, which means charging different prices based on the quantity consumed, and third degree price discrimination, which is the act of setting different prices for different groups of customers. The initial concern was to find out and analyze the implications of such practice in monopolistic markets. However, in most markets firms face competition. In fact, imperfect competition is the most common economic setting. For this reason academics progressively started to focus on the effects of price discrimination in competitive markets. Also, this type of studies has become increasingly relevant in recent years due to technological evolution[2]. Indeed, firms' recent ability to collect and process big data and to communicate with customers in a personalized way has created an environment in which firms have access to information about their customers and can use it to price discriminate. Due to this new reality, different types of price discrimination, such as location based price discrimination or behaviour-based price discrimination, became possible to implement. The latter is the subject of this dissertation and consists in setting different prices based on the consumers' past behaviour. For instance, a firm can set a lower price to historic rivals' customers than to its own loyal customers in order to poach them. Customer poaching means "capturing" the rivals' customers, making them choose to buy your product instead. Since technology has made this kind of practice increasingly easier to implement, it comes as no surprise that literature on the topic is vast and diverse.

Chen (1997) and Fudenberg and Tirole (2000) are the two seminal papers that started this wave of studies. Both papers focus on the implications of behaviour-based price discrimination in duopolistic markets in the context of a dynamic model with two periods. However, their approach is quite different. In Chen (1997), consumers face

[2] Executive Office of the President of the United States, Council of Economic Advisers (2015)

switching costs when they change their supplier. This is a key ingredient of this model since in equilibrium consumers' poaching is actually driven by switching costs. Differently, in Fudenberg and Tirole (2000) poaching is information-based: firms offer horizontally differentiated products and consumers have a preference for one of the two firms. In this model, price discrimination can only happen in the second period because firms need the first period to get to know the consumers' behaviour and, therefore, their preferences.

According to Chen (1997), higher switching costs result in more price discrimination in equilibrium. A mature market is more competitive when price discrimination is possible, but even under this condition prices increase with the expected switching costs. Under this practice, firms are worse-off and consumers are not always better-off. There is a dead-weight loss for society due to inefficient switching. The results obtained in Fudenberg and Tirole (2000) are slightly different. In an attempt to poach from rival companies, each firm sets a lower price in the second period than the one that would be set if only uniform pricing strategies were possible. However, the opposite happens in the first period. Assuming consumers are not myopic, they understand prices will decrease in time and their demand becomes less elastic. This results in higher prices in the first period. This model was extended to an infinite horizon in Villas-Boas (1999) and the conclusions regarding prices remained true. Consumers were proven to benefit from behaviour-based price discrimination while the opposite happened to firms. Society as a whole is worse-off. These three papers come to the same conclusion regarding welfare. However, there are dynamic models in which society is better-off with price discrimination. For instance, in Esteves (2010) that is the case. The main difference between this paper and both Fudenberg and Tirole (2000) and Villas-Boas (1999) lies on the distribution of consumers' tastes, since Esteves (2010) considers that the distribution of consumer types is discrete (introducing price discrimination in the set-up proposed by Shilony (1977)). The conclusions regarding prices are more in agreement with the ones found under the switching costs approach than the information-based one.

The models mentioned so far are all dynamic, but there are also papers using static models to study behaviour-based price discrimination. For example, Corts (1998) presents a duopoly model with vertically differentiated firms and two different groups of consumers. A distinction is made between best-response symmetry and best-response asymmetry. A market exhibits best-response symmetry when the strong and weak segments of each firm coincide, while best-response asymmetry is present when one firm's strong market is the other firm's weak segment[3]. In the first case, Corts (1998) obtains that the uniform price lies between the two discriminatory prices and the welfare effects are ambiguous. In the second case, one of two situations may occur: all prices may fall or all prices may increase when firms adopt price discrimination strategies. If all prices fall, it is found that competitive price discrimination may intensify competition and lead to lower prices. In this case, there is a prisoner dilemma situation in equilibrium, with firms' profits decreasing under price discrimination. This can constitute an incentive to stop the competitive unprofitable behaviour and commit to uniform pricing. When all prices increase, the opposite happens and consumers are worse-off with price discrimination, while firms are better-off.

Armstrong and Vickers (2001) study a duopoly model with heterogeneous consumers (different transportation costs). It is shown that price discrimination tends to be desirable in sufficiently competitive conditions. In this situation, total welfare increases when price discrimination is possible. However, when consumers are divided in identifiable separate markets both consumer surplus and total welfare suffer from this type of practice. In fact, only profits increase. One of these authors presented another paper, Armstrong (2009), focused on three different aspects: the information about customers available to firms; the instruments firms can use in the design of their tariffs; and the ability of firms to commit to their pricing plans. This paper shows that with competition, the effects of price discrimination on profit, consumer surplus and overall welfare depend on the information and on the tariff instruments available to firms. It is possible that the ability to commit to prices damages industry profit.

Two other very important models to study the implications of price discrimination are Shaffer and Zhang (2000) and Thisse and Vives (1988). Both the studies have slightly different approaches from the models above. For instance, Shaffer and Zhang (2000) propose a duopoly model with two unequal sized groups of consumers. When demand is symmetric, it is optimal to charge a lower price to a rival's customer. When demand is

[3] Note that the best response asymmetry set-up was also considered in Thisse and Vives (1988), which is mentioned below.

asymmetric, it may happen that a firm offers a discount to its own customers and that price discrimination lessens competition instead of intensifying it. The conclusion is that a firm should charge a lower price to the more price-elastic consumer group, whether it is the competitor's customers or not. Also, the more price-elastic group does not have to be the same for both firms. Thisse and Vives (1988) have an even more different approach. They present a duopoly model in which firms and consumers are located in a n-dimensional space and price discrimination is based on the consumer's location. The conclusion is that there is a strong tendency for firms to choose a discriminatory policy, despite the fact that they may end up worse than if they had chosen uniform pricing (prisoner dilemma outcome). Prices faced by consumers are lower than with uniform pricing and welfare is not affected.

Differently, Esteves (2009) obtained that behaviour-based price discrimination can be profitable, corroborating Armstrong and Vickers (2001). Using a duopoly model where consumers have both different product preferences and brand loyalty degree, the author studied the impact of price discrimination in firms' welfare when they only have information about one of the two differences between customers. Price discrimination was found to be profitable when firms have access to the right information but remain ignorant about the rest. Therefore, Esteves (2009) concluded that partial information can be the key factor for avoiding the prisoners' dilemma when implementing behaviour-based price discrimination.

The following table (table 1) summarizes and compares the conclusions of the previous papers. A minus sign means a smaller value when there is price discrimination in comparison to a situation in which prices are uniform. A plus sign means a higher value with price discrimination than in a context of uniform pricing. "NA" means there is no information available and a question mark represents an ambiguous situation.

<table>
<tr><th></th><th>Paper</th><th colspan="2">Prices</th><th>Consumer Surplus</th><th>Profits</th><th>Welfare</th></tr>
<tr><td rowspan="5">Static Models</td><td>Thisse and Vives (1998)</td><td colspan="2">-</td><td>NA</td><td>-</td><td>No effect</td></tr>
<tr><td>Corts (1998)[4]</td><td colspan="2">-</td><td>+</td><td>-</td><td>+
(if the output increases)</td></tr>
<tr><td>Shaffer and Zhang (2000)[5]</td><td colspan="2">-</td><td>+</td><td>-</td><td>NA</td></tr>
<tr><td>Armstrong and Vickers (2001)</td><td colspan="2">-</td><td>+</td><td>+</td><td>+</td></tr>
<tr><td>Esteves (2009)[6]</td><td colspan="2">NA</td><td>NA</td><td>+</td><td>NA</td></tr>
<tr><td></td><td></td><td>1st period</td><td>2nd period</td><td></td><td></td><td></td></tr>
<tr><td>Dynamic Model (switching costs)</td><td>Chen (1997)</td><td>-</td><td>-</td><td>?</td><td>-</td><td>-</td></tr>
<tr><td rowspan="3">Dynamic Models (brand preferences)</td><td>Fundenberg and Tirole (2000)</td><td>+</td><td>-</td><td>NA</td><td>NA</td><td>-</td></tr>
<tr><td>Villas-Boas (1999)</td><td>+</td><td>-</td><td>+</td><td>-</td><td>-</td></tr>
<tr><td>Esteves (2010)</td><td>-</td><td>-</td><td>+</td><td>-</td><td>+</td></tr>
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Table 1 – Summary of the conclusions found in the literature regarding behavior-based price discrimination in markets with symmetric firms

So far we have focused on duopoly models where both firms are symmetric and, therefore, have the same ability to price discriminate. However, reality is not necessarily like that. In many markets, there are dominant firms. For this reason it is important to verify to what extent dominant firms can use behaviour-based price discrimination to reinforce their dominant position. It is also relevant to analyze if this type of practice can constitute a barrier to entry. Both of these issues raise awareness to the possible necessity of antitrust legislation to limit or forbid firms with higher market power to engage in this type of pricing strategies.

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[4] These conclusions are only valid when there is best-response asymmetry (in the case in which all prices fall).

[5] These conclusions are only valid when demand is symmetric.

[6] This is only true when firms have access to the right information but remain ignorant about the rest.

Armstrong and Vickers (1993) is a first study about this subject. It is based on a one-period model with a dominant firm serving a monopolistic segment and competing with an entrant in a competitive segment. An asymmetric no-discrimination constraint across segments is shown to lead to a lower price in the monopolistic segment and to higher prices in the competitive one, when the entry occurs. These results may not seem beneficial for society if the competitive segment is big. However, when there are no constraints to price discrimination, there is generally less entry. This can justify the existence of constraints on price discrimination strategies.

In Chen (2008) it is analyzed a similar model but in a dynamic setting. An incumbent has a monopolistic position in one segment and competes with a more efficient firm in another segment. Price discrimination raises prices for the monopolistic segment in both periods and raises prices in the competitive segment in the first period but lowers them in the following periods. The overall effect is beneficial for the consumers as long as the more efficient firm is not forced to exit the market. When that happens, there seems to be reason for authorities to act since the trade-off between present price reductions and future price increases will harm consumers. Bouckaert *et al.* (2013) also consider a model with an incumbent and a rival, but only with two periods. Two types of asymmetric no-discrimination constraints are incorporated in the model: asymmetric no-discrimination constraints within the competitive segment (the dominant firm cannot practice behaviour-based price discrimination in the competitive segment in order to poach the rivals' customers) and asymmetric no-discrimination constraints across segments (the dominant firm cannot set a different price for the monopolistic segment and for the competitive segment). The first type of constraint lowers prices to both firms while the second type of constraint increases them. When the monopolistic segment is large enough, the effect of the constraint across segments outweighs the effect of the constraint within the competitive segment, which means that profits increase. However, the profits of the dominant firm increase when the effect of the constraint across segments is less pronounced. Both firms' profits suffer most when the monopolistic segment is small. The asymmetric no-discrimination constraint increases total welfare as well as consumer welfare when the monopolistic segment is not too large and entry is profitable. When the monopolistic segment is large, the results are more in line with the ones from Armstrong and Vickers (1993).

Mahmood and Vulkan (2013) study a similar two-period model, in which firms can only price discriminate in the second period. The main difference towards all the models mentioned until this point is that this is not a duopoly model. In fact, one of the objectives of this paper is to understand how price behaviour varies with the number of firms operating in the market. The conclusion is that the higher the number of firms, the less likely it is to price discriminate. The other objective is similar to what the previous models have focused on: study the relation between pricing behaviour and market dominance. According to this paper, in asymmetric markets, large firms are more likely to extract surplus from existing customers and offer discounts the new customers, whereas small firms tend to offer similar prices. The difference in prices between existing and new customers is on average greater for symmetric duopoly firms than to the dominant firm in the asymmetric scenario, which contradicts economic theory and common sense. Therefore, existing customers might be more disadvantaged in markets with equal sized competitors. Behaviour-based price discrimination is unlikely to raise antitrust concerns as markets become more competitive and consumers benefit from lower prices. This suggests that encouraging entry might be a better policy than regulating dominant firms.

Contrary to the previous models, Carroni (2016) argues that, when firms are asymmetric and consumers are less sophisticated than firms (less forward-looking), behaviour-based price discrimination may weaken price competition and be detrimental to consumers. This is a two-period duopoly model, similar to Bouckaert *et al.* (2013). However, in this case, the dominance of a firm is defined by the consumers' willingness to pay a price premium for their product. In this context, new conclusions arise. It is shown that, under big asymmetries between firms, the strong firm trades off current market share for future market share, while the weak firm adapts to that strategy by doing the opposite. This means, on one hand, that the strong firm accommodates the weak firm and sets high prices, enjoying high margins in spite of their relatively small market share. On the other hand, the weak firm enjoys the lessened competition in the first period and achieves a large market share, but is unable to compete in the second period. The strong firm poaches many consumers, if not all of them. In any case, both firms benefit from this strategy, even if the weak firm is forced to abandon the market. When consumers are myopic, this behaviour may result in a somehow collusive pricing behaviour. This possibility raises concerns regarding the amount of information about the customers that companies should have access to.

Gehrig *et al.* (2011) study a slightly different situation. In this static duopoly model, one firm can price discriminate while the other cannot. However, this is not a result of any constraint imposed by antitrust authorities. In this model, there is an incumbent firm - that used to be a monopolist - and an entrant. The incumbent already knows the clients and can distinguish between former clients and new clients. The same does not hold true for the entrant. Therefore, the practice of behaviour-based price discrimination is only possible for the incumbent. The objective of this paper is to analyze whether or not this information asymmetry between the firms can represent a barrier to entry and if antitrust authorities have a reason to limit price discrimination in markets such as this one. It was found that consumers loyal to the incumbent are better off with uniform pricing, whereas new consumers are better off with price discrimination. Consumer surplus is higher under uniform pricing. Also, history-based price discrimination does not encourage nor discourage market entry and, therefore, it does not affect the persistence of the incumbent's market dominance. This implies that the potential abuse of market dominance caused by history-based price discrimination is exploitation and not exclusion. A ban on price discrimination would benefit consumers, but would not interfere in the decision to enter the market. Gehrig *et al.* (2012) compares this model to a similar one in which both firms have access to information about the consumers' past behaviour (although ignoring the existence of new consumers in the market). It also compares the new model with an uniform pricing setting. This analysis shows that it is beneficial for consumers that both companies are able apply behaviour-based price discrimination. In this case, consumer surplus increases as long as the switching costs are not too high and the inherited degree of dominance is not too weak. My work will also be based on a comparison with the model presented by Gehrig *et al.* (2011). However, instead of comparing it just to a model without information asymmetry, I will change some of the assumptions. I will not have a model with an entrant. Instead there will be two different set-ups: one with a persistent information asymmetry caused by the two firms not entering the market at the same time; and another without information asymmetry for comparison purposes (which can be interpreted as the situation that would exist in the market after the last firm to enter the market was able to catch up with the level of information gathered by the incumbent). Considering the existence of previous competition between the firms, allows me to take into consideration the consumers past preferences and check how that alters the pricing decisions and, consequently, how it influences the profitability of the firms and the consumers' welfare.

3. The model

This model is an extension of Gehrig *et al.* (2011)'s model that assumes previous competition between the firms.

Consider a horizontally differentiated Hotelling market with full coverage where two firms, A and B, compete on prices in a one-period setting. The firms are located at the opposite extremes of the Hotelling line [0,1]. Firm A is located on the left side of the interval and firm B is located on the right side of it. While on Gehrig *et al.* (2011), firm A is assumed have been a monopolist in the past, that faces competition from firm B for the first time, here both firms were already operating in the market. Thus, there is at least one competitive period before our analysis starts. For this reason, there is an important variable added to our model, that corresponds to firm A's loyal market segment – x_1.

For simplicity, assume that the consumers are uniformly distributed on the interval [0,1]. Also, assume a proportion θ of the consumers is replaced by new consumers, who are also distributed uniformly on [0,1]. Because they are new to the market, no firm has information about them, creating a new dynamic channel of strategic interaction which could not be captured in the static model by Gehrig *et al.* (2011). The remaining customers are also uniformly distributed. All consumers are distributed in the unit interval according to increased preference for brand B. Each customer buys one unit of the product from one of the firms. Old customers face switching costs, which means firms need to create incentives to be able to capture their rival's clients, namely through behaviour-based price discrimination. Marginal production costs are assumed to be equal for both firms.

To study the impacts of information asymmetry, let us assume firm A has some sort of advantage over firm B in terms of the ability to gather information about the clients' behaviour – a technological advantage. There are various possible explanations for such situation, some of which as simple as having different budgets to invest in this type of technology. However, for comparison reasons, perhaps the easiest explanation is considering firm A to be a long installed firm in the market and firm B a relatively new one. This approximates the present model to Gehrig *et al.* (2011). For instance, it could represent a follow-up of that model (for example, a second period).

Two scenarios will be considered: one in which only the incumbent firm can price discriminate (it takes time until the new firm is able to match the information the incumbent firm already has); and one in which, after observing what happened in the past, both firms have enough knowledge to segment the market and price discriminate accordingly (the information asymmetry does not last). The market is divided into three categories of customers: the old customers who bought the firm's product in the first period, those who chose the rival's product in the first period and the new customers entering the market only in the second period. When firms have the ability to segment their clients into these categories, behaviour-based price discrimination emerges as a strategy to attract more customers and to poach the rival's customers.

I will use backward induction to determine the Nash equilibrium of the game. Then, I will analyze the results in order to study the impact of the initial information asymmetry on the pricing strategies and its consequences. This will allow me to understand to what extent this asymmetry influences the results of the firms and impacts consumers. At the same time, I will check what changes when past decisions are taken into consideration in the pricing strategies.

3.1. Asymmetric behaviour-based pricing

In this scenario, firm A is able to separate “old” and “new” consumers in the market. Moreover, it is also able to identify the consumers who chose each of the brands in the previous period. As a result, it is possible for firm A to segment its clients and price discriminate accordingly, meaning behaviour-based price discrimination is an option. The prices set by firm A are $\mathrm{p_a}$, $\mathrm{q_a}$ and $\mathrm{z_a}$ for the consumers who chose brand A in both periods, those who switch from brand A to B in the second period and new customers, respectively. On the other hand, firm B has no way of knowing the clients’ past behaviour. As a result, firm B sets the price $\mathrm{p_b}$ to all consumers.

Old customers face an exogenous switching cost σ when they switch from one brand to the other. The transportation cost faced by each customer is represented by τ. β is the benefit for each customer of consuming the product they chose to purchase and it is assumed to be high enough to guarantee that the market is covered in all periods.

The utility function of an old consumer who has purchased a unit of the product of firm A in the previous period indexed by *x* is then:

$$\mathrm{U}(x) = \begin{cases} \beta - \mathrm{p_a} - \tau x & \text{if continues to purchase brand A} \\ \beta - \mathrm{p_b} - \tau(1 - x) - \sigma & \text{if switches to brand B} \end{cases} \tag{1}$$

Let x_a represent a consumer, who previously preferred brand A and is now indifferent between purchasing A and B. As this consumer chose A in the previous period, he is located on the interval $[0, x_1]$, assuming preferences remain the same. This requires $\beta - \mathrm{p_a} - \tau x_\mathrm{a} = \beta - \mathrm{p_b} - \tau(1 - x_\mathrm{a}) - \sigma$ to be true for this consumer. Therefore,

$$x_\mathrm{a} = \frac{1}{2} + \frac{\sigma + \mathrm{p_b} - \mathrm{p_a}}{2\tau}\ , \ \ x_\mathrm{a} \in [0, x_1] \tag{1.1}$$

Recall that x_1 is the consumer that, in the past, was indifferent between A and B.

The utility function of an old consumer who has purchased a unit of the product of firm B in the previous period is:

$$\mathrm{U}(x) = \begin{cases} \beta - \mathrm{p_b} - \tau(1 - x) & \text{if continues to purchase brand B} \\ \beta - \mathrm{q_a} - \tau x - \sigma & \text{if switches to brand A} \end{cases} \tag{2}$$

Let x_{b} represent a consumer, who previously preferred brand B and is now indifferent between purchasing A and B. Assuming the preferences of a consumer do not change from one period to the other, we know a consumer who bought B in the past is located on the interval $[x_1, 1]$. Accordingly, for the indifferent consumer, it must be the case that $\beta - \mathrm{p_b} - \tau(1 - \mathrm{x_b}) = \beta - \mathrm{q_a} - \tau\, x_{\mathrm{b}} - \sigma$, so that:

$$x_{\mathrm{b}} = \frac{1}{2} + \frac{\mathrm{p_b} - \mathrm{q_a} - \sigma}{2\tau}, \quad x_{\mathrm{b}} \in [x_1, 1] \tag{2.1}$$

The utility function of a new consumer indexed by *x* is:

$$\mathrm{U}(x) = \begin{cases} \beta - \mathrm{z_a} - \tau x & \text{if purchases brand A} \\ \beta - \mathrm{p_b} - \tau(1 - x) & \text{if purchases brand B} \end{cases} \tag{3}$$

Let x_{n} represent a new consumer who is indifferent between purchasing A and B. This requires $\beta - \mathrm{z_a} - \tau x = \beta - \mathrm{p_b} - \tau(1 - x_{\mathrm{n}})$ to be true for this consumer. Therefore,

$$x_{\mathrm{n}} = \frac{1}{2} + \frac{\mathrm{p_b} - \mathrm{z_a}}{2\tau} \tag{3.1}$$

In order to have a meaningful model, we need to guarantee that each consumer is in the selected location. In other words, we must assure $x_b > x_a$. Plotting 1.1 and 2.1 in the previous expression, we get $x_1 > \frac{\sigma}{\tau}$ as a necessary condition. For this reason, for the rest of this model's analysis $x_1 > \frac{\sigma}{\tau}$ will be an assumption.

Assumption 1: $x_1 > \frac{\sigma}{\tau}$

The following picture represents the Hotelling lines illustrating equilibrium market segmentation in the case of old consumers (the first Hotelling line) and in the case of new consumers (the second Hotelling line).

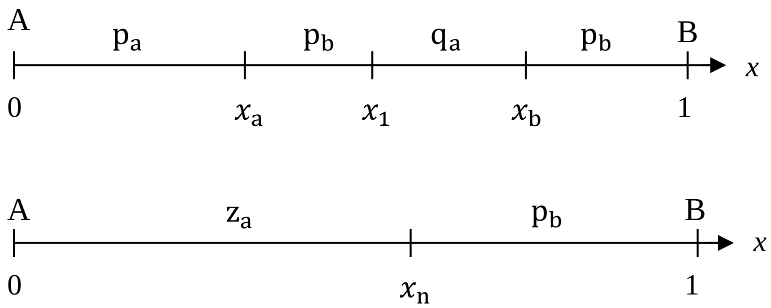

Figure 1 – Consumer allocation between horizontally differentiated brands A and B in a context with information asymmetry. Bottom: (1-θ) old consumers. Top: θ new consumers.

Since firm A is able to price discriminate, firm A chooses p_a, q_a and z_a in order to maximize its profit,

$$\pi_a = (1 - \theta)[(p_a - c)\, x_a + (q_a - c)(x_b - x_1)] + \theta(z_a - c)\, x_n$$

Instead, firm B who does not have access to the price discrimination technology (or is unable to gather information on consumers' previous choices) chooses p_b in order to maximize its profit,

$$\pi_b = (1 - \theta)[(p_b - c)((1 - x_b) + (x_1 - x_a))] + \theta(p_b - c)(1 - x_n).$$

Proposition 1: *When firm A has a privileged access to the price discrimination technology (leading to asymmetric adoption of BBPD strategies), the equilibrium prices are:*

$$p_a = c + \frac{\sigma}{2} + \frac{1}{3}\tau\frac{\theta - x_1 + \theta x_1 - 4}{\theta - 2}$$

$$p_b = c + \frac{1}{3}\tau\frac{-\theta - 2x_1 + 2\theta x_1 - 2}{\theta - 2}$$

$$q_a = c - \frac{\sigma}{2} - \frac{1}{3}\tau\frac{-\theta - 5x_1 + 2\theta x_1 + 4}{\theta - 2}$$

$$z_a = c + \frac{1}{3}\tau\frac{\theta - x_1 + \theta x_1 - 4}{\theta - 2}$$

so that the equilibrium price differential in equilibrium is equal to:

$$p_a - p_b = -\frac{1}{6(\theta-2)}(6\sigma + 4\tau - 3\theta\sigma - 4\theta\tau - 2\tau x_1 + 2\theta\tau x_1)$$

$$q_a - p_b = -\frac{1}{6(\theta-2)}(4\tau - 6\sigma + 3\theta\sigma - 4\theta\tau - 14\tau x_1 + 8\theta\tau x_1)$$

$$z_a - p_b = -\frac{1}{3}\tau\,\frac{\theta-1}{\theta-2}(x_1 - 2)$$

Proof: see the Appendix

In light of the equilibrium prices described above, Corollary 1 allows us to characterize the sub-sets of consumers who buy good A and good B. The old consumers located in $[0, x_a]$ or $[x_1, x_b]$ and the new consumers located in $[0, x_n]$ buy good A, whereas the remaining consumers end up buying good B, instead.

Corollary 1: *In a scenario of asymmetric information, with firm A having access unilateral access to the price discrimination technology (leading to asymmetric adoption of BBPD strategies), the equilibrium market segmentation is such that:*

$$x_a = \frac{1}{4}\frac{\sigma}{\tau} + \frac{1}{6}\frac{\theta - x_1 + \theta x_1 - 4}{\theta - 2}$$

$$x_b = -\frac{\sigma}{4\tau} + \frac{1}{6}\frac{\theta - 7x_1 + 4\theta x_1 - 4}{\theta - 2}$$

$$x_n = \frac{1}{6}\frac{\theta - x_1 + \theta x_1 - 4}{\theta - 2}$$

Proof: The expressions in corollary 1 can be obtained by plugging the equilibrium prices obtained in Proposition 1, in the expressions (1.1), (2.1) and (3.1).

From proposition 1, it becomes clear the price discriminating firm – firm A – sets a higher price for its old and loyal customers and a lower price to the customers they are trying to poach ($p_a - q_a = \sigma + \tau x_1$). This is the expected result based on the existent literature[7].

In addition, the consumers' past behaviour affects the price decisions. For instance, an increase in x_1 has a positive impact on the prices set by firm A to its loyal customers and a negative impact on the prices for former firm B customers, as expected. With a higher x_1, the segment of firm A's loyal customers increases and allows the firm to practice higher prices without losing clients and, at the same time, attract the rival's clients by setting a lower price for them. Moreover, x_1 has a positive impact on the price set by the incumbent firm on the segment of the new customers and on firm B's uniform price. Facing a strong segment of clients loyal to firm A and without the possibility of price discriminating, firm B faces a difficult situation (given the rival's ability to set low prices to poach clients who previously preferred B and to gather clients on the segment of new consumers). Unable to enter a price war, firm B opts for increasing prices in an attempt to profit. Otherwise, firm B would have to set a low price to try to compete with firm A's ability to price discriminate. Having a competitive disadvantage towards its rival, firm B would risk losing clients and potentially not gaining enough new ones to profit. Also, the fact that firm A softens its price strategy towards loyal clients might work

[7] See, for instance, Chen (2005), Shaffer and Zhang (2000) and Gehrig et al (2011).

as an incentive for firm B to set higher prices for those clients. As the firm's price is uniform, all segments of the market are affected by this decision.

The impacts of a change of the transportation costs are a little more complex. In the new segment, an increase in τ generates an increase of z_a if (θ - x_1+ θx_1 - 4) < 0 and a decrease otherwise. For the old customers, an increase in τ means: an increase in p_a if (θ - x_1 + θx_1 - 4) < 0 and a decrease otherwise; an increase in p_b if (θ + $2x_1$- $2\theta x_1$ + 2) > 0 and a decrease otherwise (since x_1< 1, the expression is always positive); and, finally, an increase in q_a if (θ + $5x_1$ - $2\theta x_1$ - 4) < 0 and a decrease otherwise. When we restrict θ and x_1 to the interval of possible values the conclusions become simpler than it initially seemed. For instance, in line with standard Hotelling models we obtain that p_a, z_a and p_b always increase with τ. However, q_a can decrease with an increase of τ, though that is not the case for most values. In fact, it only happens when firm A has a very strong inherited dominance (x_1 higher than 0.8), especially when the percentage of new consumers is small.

Figure 2 below depicts the impact of the transportation cost on the degree of product differentiation. In the "+" area, prices targeted to the rival's old customers are increasing with τ, whereas in the "-" area, they are decreasing with τ.[8]

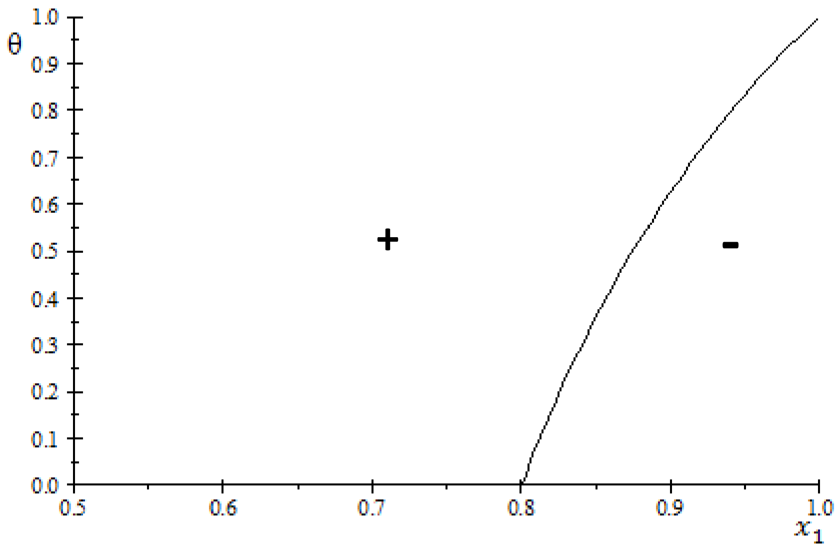

Figure 2 - Impact of τ on the price set by firm A to the poached clients (q_a).

[8] The domain is restricted to x_1 > 0.5 due to the assumption $x_1 > \frac{\sigma}{\tau}$.

A possible explanation for these results is that an increase on transportation costs leads firm A to increase prices to its loyal segment and, to benefit from it, firm B increases its price as well. Also, a high percentage of new consumers in the market (which implies a smaller percentage of old consumers) helps lessening the competition in the old segment. So, for most values, all prices will be higher for old customers with an increase of the transportation costs. However, when x_1 is high enough and there are not many new customers (firm A has a very strong position in the market and wants to maintain it), the possibility of a future entrance of new customers might be perceived as a threat to that position, making the dominant firm act in order to poach its rival's previous clients. In fact, with a very strong base of loyal clients, firm A is able to increase the demand for its product significantly by poaching the rival's clients without risking a price war (remember that B sets an uniform price and, therefore, to benefit from the increase of p_a and given that firm A's loyal segment is big, p_b still needs to increase).

The unit increase of production costs is completely passed through to all prices in the market. This means, an unit increase on c generates an unit increase on p_a, q_a, z_a and p_b. Moreover, it is possible to conclude that all prices rise when the proportion of new consumers in the market increases. Accordingly, the model predicts that prices are higher in expanding markets than in markets in which the proportion of consumers entering the market is rather small, regardless of the previous supplier of old consumers.

Finally, switching costs only affect the prices set by the discriminating firm on the segment of old customers. When substitution costs increase, the price for loyal customers increases as well, whereas the prices for previous firm B customers decrease (since poached consumers need to be compensated for the extra switching cost they incur when they switch from one brand to the other). Thus, in the old customers' segment, the existence of switching costs allows the price-discriminating firm to set higher prices to its loyal customers at the cost of setting lower prices to the poached customers, as the literature predicts. This works as an incentive to switch firms in a context where that change becomes more costly.

Profits

From proposition 2 and the profit expressions, it is possible to find the equilibrium profits for firms A and B.

In equilibrium, the profits of each firm are:

$$\pi_a = -\frac{1}{36}\left(\frac{18\sigma^2+32\tau^2+9\theta^2\sigma^2+2\theta^2\tau^2+26\tau^2 x_1{}^2-27\theta\sigma^2-16\theta\tau^2-32\tau^2 x_1+8\theta^2\tau^2 x1^2}{\tau(\theta-2)} + \frac{40\theta\tau^2 x_1-34\theta\tau^2 x_1{}^2-88\theta^2\tau^2 x_1+36\sigma\tau x_1+18\theta^2\sigma\tau x_1-54\theta\sigma\tau x_1}{\tau(\theta-2)}\right)$$

$$\pi_b = -\frac{1}{18}\tau\frac{(-\theta-2x_1+2\theta x_1-2)^2}{\theta-2}$$

We will now do a comparative static analysis in order to identify the impact of the different exogenous variables on the firms' equilibrium profits, as we previously did with the prices. The effect of x_1 on the profits is not necessarily the same for every value of that variable.

$$\frac{d}{dx1}(\pi a) = -\frac{\theta-1}{18\theta-36}(16\tau - 18\sigma + 9\theta\sigma - 4\theta\tau - 26\tau x_1 + 8\theta\tau x_1) \tag{4.1}$$

$$\frac{d}{dx1}(\pi b) = \frac{1}{9}\frac{\tau}{\theta-2}(2\theta-2)(\theta+2x_1-2\theta x_1+2) \tag{4.2}$$

From (4.1) it is possible to conclude that the impact of an increase in x_1 over the profits of firm A is positive if $(16\tau - 18\sigma + 9\theta\sigma - 4\theta\tau - 26\tau x_1 + 8\theta\tau x_1) < 0$ and negative otherwise. From (4.2) it is clear the effect of x_1 on the profits of the firm that does not have the ability to price discriminate is positive if $(\theta + 2x_1 - 2\theta x_1 + 2) > 0$ and negative otherwise. Given that the percentage of new clients is necessarily inferior to one, we have that the profits of the non-discriminating firm are always positively influenced by the position of x_1.

Assuming, for simplification, $\tau = 1$ and $\sigma = 0.5$, we have, as expected, that same result. The same conclusion is true for the price-discriminating firm. Thus, under our assumptions, if firm A's loyal base of clients increases, so do the profits of both firms.

Regarding the portion of new consumers (θ), the effects over the profits are somewhat harder to analyse.

$$\frac{d}{d\theta}(\pi a) = -\frac{1}{36\tau(\sigma-2)^2}(9\theta^2\sigma^2 + 18\theta^2\sigma\tau x_1 + 8\theta^2\tau^2 {x_1}^2 - 8\theta^2\tau^2 x_1 + 2\theta^2\tau^2 - 36\theta\sigma^2 - 72\theta\sigma\tau x_1 - 32\theta\tau^2 {x_1}^2 + 32\theta\tau^2 x_1 - 8\theta\tau^2 + 36\sigma^2 + 72\sigma\tau x_1 + 42\tau^2 {x_1}^2 - 48\tau^2 x_1) \tag{5.1}$$

$$\frac{d}{d\theta}(\pi b) = \frac{1}{18}\frac{\tau}{(\theta-2)^2}(-4\theta^2 {x_1}^2 + 4\theta^2 x_1 - \theta^2 + 16\theta {x_1}^2 - 16\theta x_1 + 4\theta - 12{x_1}^2 + 12) \tag{5.2}$$

From (5.1) it is possible to observe that the impact of θ on the incumbent firm profits is positive if $(9\theta^2\sigma^2 + 18\theta^2\sigma\tau x_1 + 8\theta^2\tau^2 {x_1}^2 - 8\theta^2\tau^2 x_1 + 2\theta^2\tau^2 - 36\theta\sigma^2 - 72\theta\sigma\tau x_1 - 32\theta\tau^2 {x_1}^2 + 32\theta\tau^2 x_1 - 8\theta\tau^2 + 36\sigma^2 + 72\sigma\tau x_1 + 42\tau^2 {x_1}^2 - 48\tau^2 x_1)$ < 0 and positive otherwise. On the other hand, according to (5.2), the effect on firm B's profits is positive if $(-4\theta^2 {x_1}^2 + 4\theta^2 x_1 - \theta^2 + 16\theta {x_1}^2 - 16\theta x_1 + 4\theta - 12{x_1}^2 + 12)$ > 0 and negative otherwise.

Considering, again, τ = 1 and σ = 0.5, an increase of new consumers in the market always benefits the non-discriminating firm. However, the same is not necessarily true for the other firm. In fact, as can be seen below, for low enough values of both x_1 and θ, an addition of the proportion of new consumers in the market is harmful for firm A. Thus, there is a range of values for these variables where the entry of new customers in the market comes as a disadvantage to the firm with the ability of price discriminating. This happens because the entry of new customers in the market attenuates the advantage of having loyal customers.

In the following figure, a plus sign means the profits of firm A increase with an increase of the portion of new customers.[9]

[9] The domain is restricted to $x_1 > 0.5$ due to the assumption $x_1 > \frac{\sigma}{\tau}$.

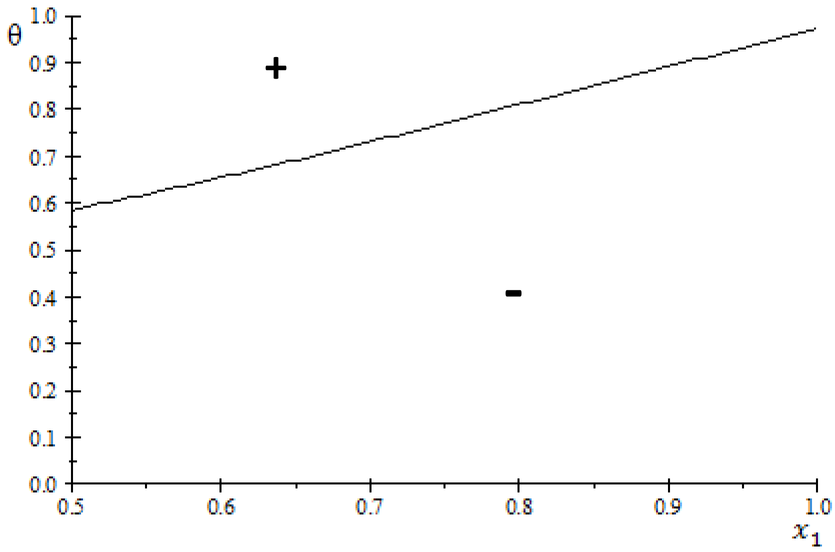

Figure 3 - Impact of θ on the profits of firm A.

It is also relevant to study the impact of the switching costs on the profits.

$$\frac{d}{d\sigma}(\pi a) = -\frac{1}{2\tau}(\theta - 1)(\sigma + \tau x_1) \quad (6.1)$$

$$\frac{d}{d\sigma}(\pi b) = 0 \quad (6.2)$$

If $(\sigma + \tau x_1) > 0$, then the impact of σ on the profits of firm A is positive. This condition is true for all possible values of x_1, which implies that an increase on the switching costs always increases the profits of the price-discriminating firm. Conversely, switching costs have no effect over the profits of the firm that does not price discriminate. The reason behind this conclusion is that there is no heterogeneity between clients regarding the switching cost (this cost is the same for all of the old clients) and firm B has no possibility of adjusting prices in order to only compensate the clients who actually switch firms. Therefore, this is not a factor the firm takes into consideration when setting a single price for the entire market.

Lastly, we will look at the transportation costs.

$$\frac{d}{d\tau}(\pi a) = \frac{1}{36\tau(\sigma-2)^2}(9\theta^2\sigma^2 - 8\theta^2\tau^2{x_1}^2 + 8\theta^2\tau^2 x_1 - 2\theta^2\tau^2 - 27\theta\sigma^2 + 34\theta\tau^2{x_1}^2 - 40\theta\tau^2 x_1 + 16\theta\tau^2 + 18\sigma^2 - 26\tau^2{x_1}^2 + 32\tau^2 x_1 - 32\tau^2) \tag{7.1}$$

$$\frac{d}{d\theta}(\pi b) = -\frac{1}{18(\theta-2)^2}(\theta + 2x_1 - 2\theta x_1 + 2)^2 \tag{7.2}$$

The impact of the transportation costs on the profits of firm A is positive if $(9\theta^2\sigma^2 - 8\theta^2\tau^2{x_1}^2 + 8\theta^2\tau^2 x_1 - 2\theta^2\tau^2 - 27\theta\sigma^2 + 34\theta\tau^2{x_1}^2 - 40\theta\tau^2 x_1 + 16\theta\tau^2 + 18\sigma^2 - 26\tau^2{x_1}^2 + 32\tau^2 x_1 - 32\tau^2) < 0$ and negative otherwise. The effect of the transportation costs on the profits of firm B is always positive. In fact, in the scenario we have been considering ($\tau = 1$ and $\sigma = 0.5$), both firms are always benefited by an increase on transportation costs.

Consumer surplus

The consumer surplus when only firm A discriminates prices is given by:

$$CS = -\frac{1}{72}\left(\frac{-144c\tau - 18\sigma^2 - 88\tau^2 + 9\theta^2\sigma^2 + 2\theta^2\tau^2 + 144\beta\tau + 26\tau^2 x_1^2 - 27\theta\sigma^2 - 4\theta\tau^2 - 80\tau^2 x_1}{\tau} + \frac{8\theta^2\tau^2 x_1^2 88\theta\tau^2 x_1 72c\theta\tau - 72\theta\beta\tau - 34\theta\tau^2 x_1^2 - 8\theta^2\tau^2 x_1 - 108\sigma\tau x_1 - 54\theta^2\sigma\tau x_1 + 162\theta\sigma\tau x_1}{\tau}\right) \tag{8}$$

The computations behind the computation of the Equilibrium CS are presented in Appendix.

3.2. Behaviour-based pricing

Let us now analyze the case in which both firms are able to distinguish old (1- θ) and new customers (θ). Also, they know which consumers chose each brand in the previous period and, as a result, their preferences. Therefore, both firms are in equal conditions to price discriminate in the second period. As in the previous model, the prices set by firm A are $\mathrm{p_a}$, $\mathrm{q_a}$ and $\mathrm{z_a}$ for the consumers who chose brand A in both periods, those who switch from brand A to B in the second period and new customers, respectively. The notation is similar for the three prices set by firm B.

The utility function of an old consumer who has purchased a unit of the product of firm A in the past indexed by *x* is:

$$\mathrm{U}(x) = \begin{cases} \beta - \mathrm{p_a} - \tau x & \text{if continues to purchase brand A} \\ \beta - \mathrm{q_b} - \tau(1-x) - \sigma & \text{if switches to brand B} \end{cases} \tag{9}$$

Let x_a represent a consumer, who previously preferred brand A, who is now indifferent between purchasing A and B. As this consumer chose A in the past, he is located on the interval $[0\ , x_1]$, assuming preferences remain the same. This requires $\beta - \mathrm{p_a} - \tau\, x_a = \beta - \mathrm{q_b} - \tau(1 - x_a) - \sigma$ to be true for this consumer. Therefore,

$$x_a = \frac{1}{2} + \frac{\sigma + \mathrm{q_b} - \mathrm{p_a}}{2\tau}\ ,\ \ x_a \in [0\ , x_1] \tag{9.1}$$

Recall that x_1 is the consumer that, in the past, was indifferent between A and B.

The utility function of an old consumer who has purchased a unit of the product of firm B in the past is:

$$\mathrm{U}(x) = \begin{cases} \beta - \mathrm{p_b} - \tau(1-x) & \text{if continues to purchase brand B} \\ \beta - \mathrm{q_a} - \tau x - \sigma & \text{if switches to brand A} \end{cases} \tag{2}$$

Let x_b represent a consumer, who previously preferred brand B and is now indifferent between purchasing A and B. With persistent preferences, we know a consumer who previously bought B in the is located on the interval $[x_1, 1]$. This requires $\beta - \mathrm{p_b} - \tau(1 - x_b) = \beta - \mathrm{q_a} - \tau x_b - \sigma$ to be true for this consumer. Therefore,

$$x_b = \frac{1}{2} + \frac{\mathrm{p_b} - \mathrm{q_a} - \sigma}{2\tau}, \; x_b \in [x_1, 1] \tag{2.1}$$

The utility function of a new consumer indexed by *x* is:

$$\mathrm{U}(x) = \begin{cases} \beta - \mathrm{z_a} - \tau x & \text{if purchases brand A} \\ \beta - \mathrm{z_b} - \tau(1-x) & \text{if purchases brand B} \end{cases} \tag{10}$$

Let x_n represent a new consumer who is indifferent between purchasing A and B. This requires $\beta - \mathrm{z_a} - \tau x_n = \beta - \mathrm{z_b} - \tau(1 - x_n)$ to be true for this consumer. Therefore,

$$x_n = \frac{1}{2} + \frac{\mathrm{z_b} - \mathrm{z_a}}{2\tau} \tag{10.1}$$

As in the asymmetric model, it is necessary to guarantee that $x_b > x_a$. Plotting 9.1 and 2.1 in the previous expression, we get $\tau > \sigma$. For this reason, for the rest of this model's analysis we will assume $\tau > \sigma$.

Assumption 2: $\tau > \sigma$

The following figure represents the Hotelling lines illustrating equilibrium market segmentation in the second period in the case of old consumers (the first Hotelling line) and in the case of new consumers (the second Hotelling line).

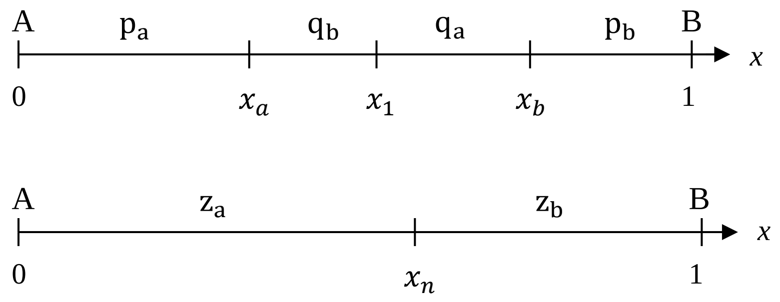

Figure 4 – Consumer allocation between horizontally differentiated brands A and B with both firms following a behaviour-based price discrimination strategy. Bottom: (1-θ) old consumers. Top: θ new consumers.

Since both firms are able to price discriminate, firm A chooses p_a, q_a and z_a in order to maximize its profit,

$$\pi_a = (1 - \theta)[(p_a - c)\, x_a + (q_a - c)(x_b - x_1)] + \theta(z_a - c)\, x_n$$

Using a similar reasoning, firm B chooses p_b, q_b and z_b in order to maximize its profit,

$$\pi_b = (1 - \theta)[(p_b - c)(1 - x_b) + (q_b - c)(x_1 - x_a)] + \theta(z_b - c)(1 - x_n).$$

Proposition 2: *When both firms have access to the price discrimination technology, the equilibrium prices are:*

$p_a = c + \frac{1}{3}\sigma + \frac{1}{3}\tau + \frac{2}{3}\tau x_1$; $p_b = c + \frac{1}{3}\sigma + \tau - \frac{2}{3}\tau x_1$; $q_a = c - \frac{1}{3}\sigma + \tau - \frac{4}{3}\tau x_1$;

$q_b = c - \frac{1}{3}\sigma - \frac{1}{3}\tau + \frac{4}{3}\tau x_1$; $z_a = z_b = c + \tau$

so that the equilibrium price differential in equilibrium is equal to:

$$p_a - p_b = \frac{2}{3}\tau\,(2x_1 - 1)$$

$$q_a - q_b = -\frac{4}{3}\tau\,(2x_1 - 1)$$

$$z_a - z_b = 0$$

Proof: See the appendix

In light of the equilibrium prices described above, Corollary 2 allows us to characterize the sub-sets of consumers who buy good A and good B. The old consumers located in $[0, x_a]$ or $[x_1, x_b]$ and the new consumers located in $[0, x_n]$ buy good A, whereas the remaining consumers end up buying good B, instead.

Corollary 2: *When both firms have access to the same information and to the price discrimination technology (leading to the adoption of BBPD strategies), the equilibrium market segmentation is such that:*

$$x_a = \frac{1}{2\tau}\left(\frac{1}{3}\sigma - \frac{2}{3}\tau + \frac{2}{3}\tau x_1\right) + \frac{1}{2}$$

$x_{\mathrm{b}} = \frac{1}{2} - \frac{1}{2\tau}\left(\frac{1}{3}\sigma - \frac{2}{3}\tau x_1\right)$

$x_{\mathrm{n}} = \frac{1}{2}$

Proof: The expressions in corollary 2 can be obtained by plugging the equilibrium prices obtained in Proposition 2, in the expressions (9.1), (2.1) and (10.1).

From Proposition 2, it is possible to conclude that in the segment of the new customers the switching costs and the behaviour in the past have no effect on the price decision. The fact that both firms are now able to price discriminate allows the firms to fully separate the markets and, therefore, in the segment of new consumers (without any previous consumption history), standard Hotelling results prevail. The same is not true for the old customers' segment. In this segment, the existence of switching costs allows the firms to set higher prices to their loyal customers at the cost of setting lower prices to the poached customers. This result corroborates the conclusions found on the existent literature, namely in papers that consider switching costs, such as Chen (2005) and Gehrig *et al* (2011). Also, the consumers' past behaviour has an impact on the price decisions. For instance, an increase in x_1 has a positive impact on the prices set to those on the left of x_1 and a negative impact on those on the right. In fact, $\mathrm{p_a}$ and $\mathrm{q_b}$ (prices to those on the left of the indifferent consumer) are only higher than $\mathrm{p_b}$ and $\mathrm{q_a}$ (prices to those on the left of the indifferent consumer), respectively, if firm A's loyal base of clients is higher than half of the old customers segment, as can be observed in the price differentials.

The impacts of a change on the transportation costs are a little more complex. In the new segment, an increase in τ always generates an increase on the prices, whose magnitude is the same for all the consumers (in fact a unit increase in the transportation cost parameter, increases prices in the new consumers segment exactly by the same amount). An increase in τ also generates an increase on the prices set to the old customers who do not change suppliers from one period to the other (loyal consumers). However, this may not be true for the poached customers. In fact, that will depend on the value of x_1. If x_1 is high enough (higher than $\frac{3}{4}$), when τ increases, $\mathrm{q_a}$ decreases, and if x_1 is low enough (lower than $\frac{1}{4}$), an increase on τ will result in a decrease on q_b. Therefore, only in the interval $[\frac{1}{4}, \frac{3}{4}]$, corresponding to a situation in which firms' asymmetry in the past was relatively mild (without strong dominance of any of the firms), will both prices simultaneously have the same behaviour as p_a, p_b, z_a and z_b. A possible explanation is that when one of the firms has a very strong inherited position, there are few customers to poach and, therefore, a decrease of the prices on that segment will not have negative consequences over the firm's profits. In this situation, the loyal customer base is seen as big enough to support all the expenses related to the increase of the costs, without choosing to switch firms.

Profits

From proposition 2 and the profit expressions, it is possible to find the equilibrium profits for firms A and B.

In equilibrium, the profits of each firm are:

$$\pi_a = -\frac{1}{18}\frac{-2\sigma^2-10\tau^2+4\sigma\tau-20\tau^2{x_1}^2+2\theta\sigma^2+\theta\tau^2+20\tau^2x_1-20\theta\tau^2x_1-4\theta\sigma\tau+20\theta\tau^2{x_1}^2-12\sigma\tau x_1+12\theta\sigma\tau x_1}{\tau}$$

$$\pi_b = -\frac{1}{18}\frac{-2\sigma^2-10\tau^2-8\sigma\tau-20\tau^2{x_1}^2+2\theta\sigma^2+\theta\tau^2+20\tau^2x_1-20\theta\tau^2x_1+8\theta\sigma\tau+20\theta\tau^2{x_1}^2+12\sigma\tau x_1-12\theta\sigma\tau x_1}{\tau}$$

We will now analyse the impact of the different variables on the firms' profits, as we previously did with the prices. The effect of x_1 on the profits is not necessarily the same for every value of that variable.

$$\frac{d}{dx1}(\pi a) = -\frac{2}{9}(\theta - 1)(3\sigma - 5\tau + 10x_1) \quad (11.1)$$

$$\frac{d}{dx1}(\pi b) = \frac{2}{9}(\theta - 1)(3\sigma + 5\tau - 10x_1) \quad (11.2)$$

From (11.1) it is possible to conclude that the impact of an increase in x_1 over the profits of firm A is positive if (3σ - 5τ + $10x_1$) > 0 and negative otherwise. From the expression, we can take that if x_1 > 0.5 τ the impact is always positive. If that condition is not true, then it is possible for the impact to be negative if 3σ < -5τ + $10x_1$. From (11.2) we can show that the same impact over the profits of firm B would only be positive if (3σ + 5τ – $10x_1$) < 0. That will never be true if x_1 < 0.5 τ. If that condition does not hold, then the impact is positive as long as 3σ < 5τ + $10x_1$. Therefore, on both cases, the conclusions depend on the transportation costs.

Considering the assumption we have been using (τ = 1 and σ = 0.5), we have that the profits of firm A increase with x_1 as long as x_1 is higher than 0.35 and θ ≠ 1. As for the profits of firm B, an increase only takes place when x_1 is higher than 0.65 and θ ≠ 1. This means that there is an area where the profits of both firms can increase simultaneously with the initial dominant position of firm A. In this case, that area is x_1 ϵ]0.65 , 1[. Therefore, we can conclude that when firm A has an inherited strong dominance, both firms would benefit from it. The exact opposite happens when A does not inherit a strong enough position. When that dominance is strong but not too strong, firm A can be the only one to benefit with x_1.

Regarding the portion of new consumers (θ), the effects over the profits are somewhat harder to analyse.

$$\frac{d}{d\theta}(\pi a) = -\frac{1}{18}\frac{2\sigma^2+\tau^2-4\sigma\tau+20\tau^2x_1(x_1-1)+12\sigma\tau x_1}{\tau} \qquad (12.1)$$

$$\frac{d}{d\theta}(\pi b) = -\frac{1}{18}\frac{2\sigma^2+\tau^2+8\sigma\tau+20\tau^2x_1(x_1-1)-12\sigma\tau x_1}{\tau} \qquad (12.2)$$

However, the difference between the two derivatives is simple and depends on the position of the indifferent consumers regarding the middle of the segment.

$$\frac{d}{d\theta}(\pi a) - \frac{d}{d\theta}(\pi b) = -\frac{2}{3}(2x_1 - 1) \qquad (12.3)$$

This shows that when firm A has a strong base of loyal clients ($x_1 > 0.5$), the entrance of new customers benefits firm B more than firm A. When $x_1 < 0.5$, the contrary is true.

It is also important to check whether or not it is possible that only one firm benefits from the appearance of new consumers. Assume for simplification that the transportation costs are equal to 1 and that the substitution costs are equal to 0.5. In this scenario, there is always a firm benefiting from an increase on θ but if x_1 is low enough or high enough there is also one firm that suffers a negative impact profit. For instance, if $x_1 < 0.26594$ only firm A benefits and the opposite happens when $x_1 > 0.73406$. There is not any location of the consumers that impacts negatively the profits of both firms simultaneously.

Furthermore, similarly to what happens with the impact of the location of the indifferent consumer, the impact of the substitution costs on the profits depends on the value of x_1.

$$\frac{d}{d\sigma}(\pi a) = -\frac{2}{9\tau}(\theta - 1)(\sigma - \tau + 3\tau x_1) \tag{13.1}$$

$$\frac{d}{d\sigma}(\pi b) = -\frac{2}{9\tau}(\theta - 1)(\sigma + 2\tau - 3\tau x_1) \tag{13.2}$$

If $(\sigma - \tau + 3\tau x_1) > 0$, then the impact of σ on the profits of firm A is positive. The impact will always be positive if $x_1 > \frac{1}{3}$. If $(\sigma + 2\tau - 3\tau x_1) > 0$, the impact of σ on the profits of firm B is positive.

Considering again that $\tau = 1$ and $\sigma = 0.5$, the profits of firm A increase with σ as long as x_1 is higher than $\frac{1}{6}$ and $\theta \neq 1$. On the other hand, firm B's profits only increase when x_1 is less than $\frac{5}{6}$ and $\theta \neq 1$. This implies that there is an area where the profits of both firms can increase simultaneously with the switching costs. In this case, that area is $x_1 \in]\frac{1}{6}, \frac{5}{6}[$. Therefore, we can conclude that for both firms to benefit from an increase on the substitution costs, the market share of the firms in the previous period can't be too high. If that happens, only the firm covering most of the market is benefited, which means that switching costs may hinder the expansion of smaller firms in the market, even when new consumers are entering the market. Indeed, for the smaller firm, it becomes hard to poach clients and, therefore, to raise its small market share, which remains small.

Lastly, the effect of the transportation costs over the profits is the same for both firms. In our range of possible values for the variables, even if transportation costs may have a negative impact on the prices targeted to non-loyal old consumers, the overall effect of transportation costs on profits is always positive, which means both firms benefit from an increase of the transportation costs, in line with standard Hotelling models .

Consumer surplus

The consumer surplus when both firms price discriminate is given by the expression:

$$CS = -\frac{1}{36}\left(\frac{36c\tau-2\sigma^2+44\tau^2+16\sigma\tau-36\beta\tau+52\tau^2x_1^2+2\theta\sigma^2+\theta\tau^2-52\tau^2x_1+52\theta\tau^2x_1-16\theta\sigma\tau}{\tau}+\frac{-52\theta\tau^2x_1^2}{\tau}\right) \quad (14)$$

The computations behind the computation of the Equilibrium CS are presented in Appendix.

3.3. Comparative analysis

In this section, we will compare the two scenarios previously considered, in terms of prices and its consequences to society. For simplification, we will assume $\tau = 1$ and $\sigma = 0.5$, like we did before, as well the inexistence of marginal production costs for both firms (c=0).

The only absolute conclusion that we can draw from the analysis of propositions 1 and 2 is that new consumers to the market always pay a higher price when there is behaviour-based price discrimination. All others may pay higher or lower prices depending of both their past behaviour and the percentage of new consumers. For instance, the loyal customers of firm A and the clients poached by firm B (this group corresponds to the former firm A clients) pay a lower price with behaviour-based price discrimination unless x_1 is very high and θ is low. When the loyal segment of firm A is small, the rival is not too interested in poaching strategies (firm B is uninterested in creating a special price – a lower price – to capture such a small group of clients, especially given the existence of switching costs that would aggravate the discount on the price). This reduces price competition. The same happens for big values of θ because a big segment of new customers implies a small segment of old customers and, hereby, a small loyal segment for firm A. On the other hand, clients poached by firm A and loyal firm B customers (former firm B clients) pay a higher price unless x_1 and θ are high enough. A high x_1 means a small loyal customer base for firm B. This relaxes competition on prices, since firm A has small interest in poaching those clients. With a high percentage of new clients, this interest is also reduced. The following graphic representations help clarifying these conclusions. A plus sign represents a higher price in the symmetric behaviour-based pricing model as opposition to the asymmetric behaviour-based pricing model[10].

[10] The domain in all the following figures is restricted to $x_1 > 0.5$ because the asymmetric behavior-based pricing model is only valid with the assumption $x_1 > \frac{\sigma}{\tau}$.

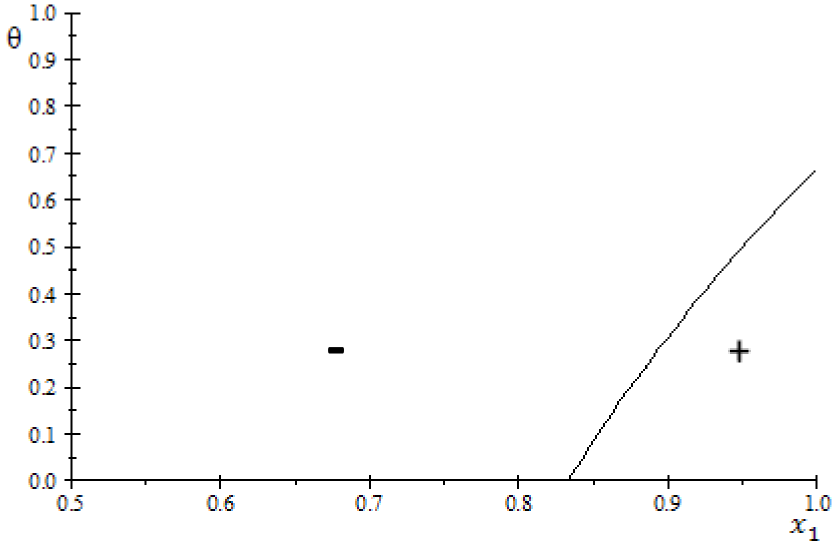

Figure 5 – Sign of the difference between the price faced by loyal firm A clients in the behaviour-based pricing model and the asymmetric behaviour-based pricing model.

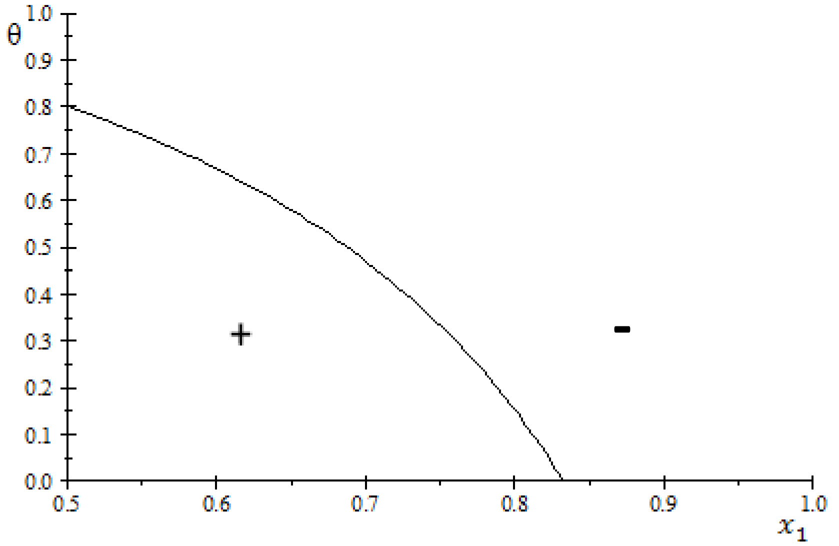

Figure 6 – Sign of the difference between the price faced by the clients poached by firm A in the behaviour-based pricing model and the asymmetric behaviour-based pricing model.

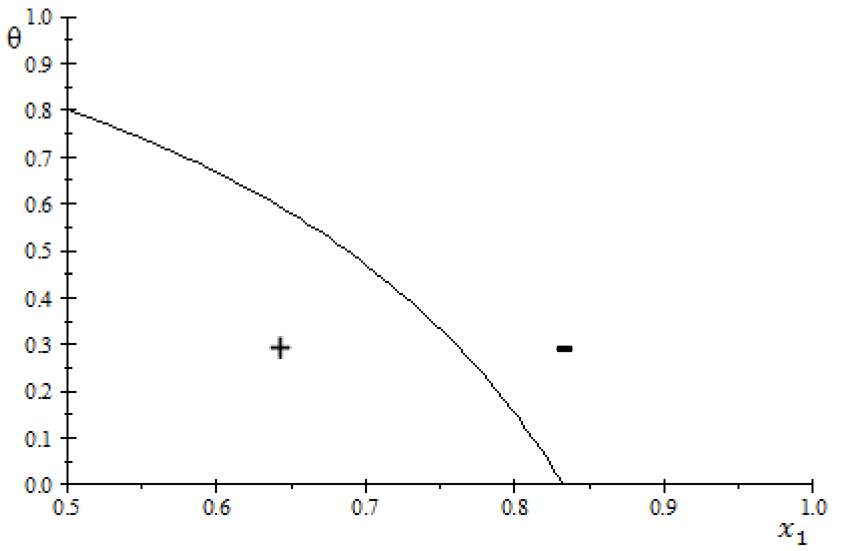

Figure 7 – Sign of the difference between the price faced by loyal firm B clients in the behaviour-based pricing model and the asymmetric behaviour-based pricing model.

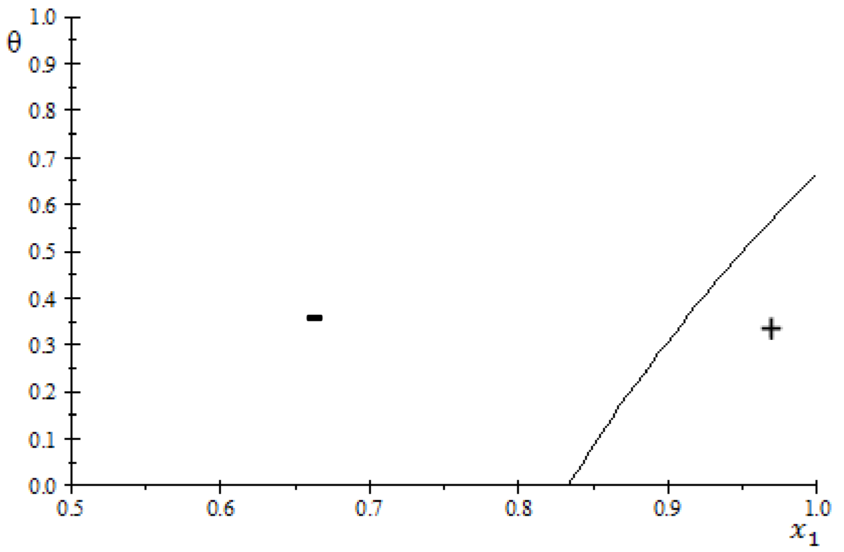

Figure 8 – Sign of the difference between the price faced by the clients poached by firm B in the behaviour-based pricing model and the asymmetric behaviour-based pricing model.

Regarding the profits, it is not surprising that firm B is benefitted when able to also price discriminate, as it is able to respond to the rival's strategy using a similar strategy. On the other hand, firm A's profits can either increase or decrease when firm B is also able to price discriminate (in comparison to the case where only firm A is able to price discriminate). In most cases, firm B's ability to price discriminate results in a decrease of the profits of firm A, compared to the situation where its rival was not able to separate clients. However, for high enough values of x_1 or θ the profits increase. In fact, with a high percentage of new clients in the market, firm A can benefit from the higher prices those clients pay (in comparison to the situation when only firm A price discriminates). Also, the existence of a strong base of clients loyal to firm A can result, as mentioned above, in less competition in that segment, allowing firm A to profit more than if firm B was unable to price discriminate. In the following figures, a plus sign represents higher profits in the symmetric behaviour-based pricing model as opposition to the asymmetric behaviour-based pricing model.

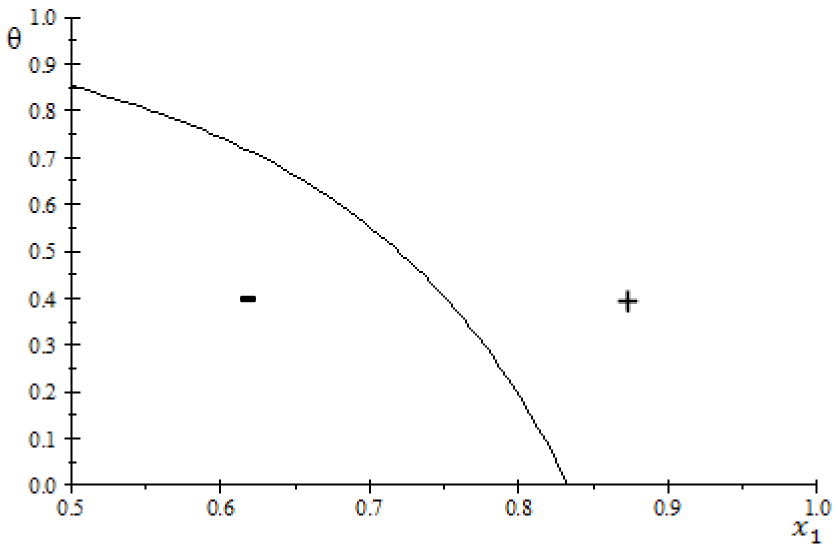

Figure 9 – Sign of the difference between the profits of firm A in the behaviour-based pricing model and the asymmetric behaviour-based pricing model.

Moreover, consumers can also be better off or worse off when both firms segment clients and price discriminate accordingly. Given the assumptions made about τ, σ and c, consumers benefit from this situation in the interval below.

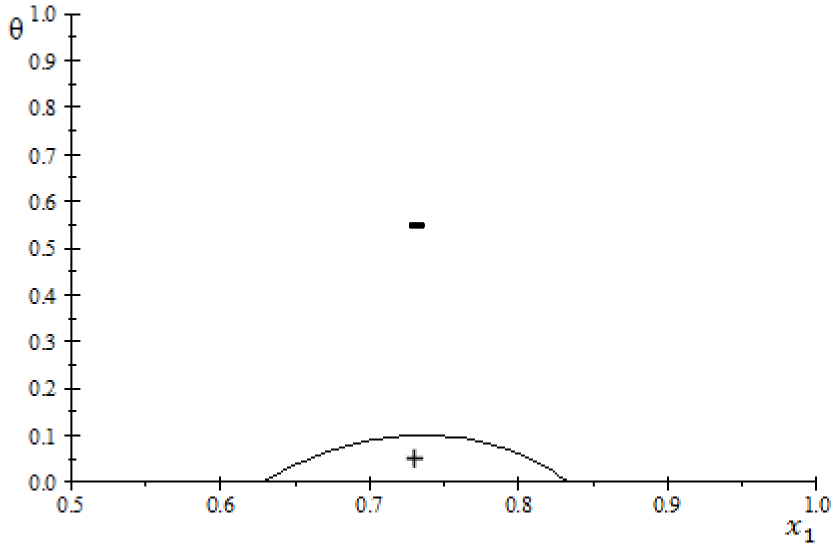

Figure 10 – Sign of the difference between the consumer surplus in the behaviour-based pricing model and the asymmetric behaviour-based pricing model.

Proof: see Appendix

3.4. Comparative analysis of the welfare consequences between our model and Gehrig *et al.* (2011)

Gehrig *et al.* (2011) studies a static duopoly model in which only one of the firms is able to price discriminate. There is an incumbent firm, which used to be a monopolist, and an entrant. The entrant is still unable to access enough information about the clients to segment them. On the other hand, the incumbent firm can distinguish between the clients who were already in the market and those who are new to the market. This information asymmetry implies that, while the incumbent firm is able to price discriminate between the two types of clients, the entrant has to set a single price for all of them.

Our model considers a similar situation. However, although our model only has one period of strategic competition, firms are confronted with past choices. It is, therefore, assumed the existence of previous competition between both firms.[11] Thus, we can compare the consequences of the equilibrium prices of both models to consumers and firms. For this comparison, we will consider the scenarios 3.1 and 3.2. This means we will compare both the situation in which the information asymmetry remains over time and the situation in which that asymmetry only lasts one period with the conclusions of Gehrig *et al.* (2011). Recall that in Gehrig *et al.* (2011) it is not considered the possibility of the entrant firm being able to override the gap in terms of information. This is will give us a wider view of the possible impacts of the information asymmetry to society, given that the results depend on the decisions made in the past. Therefore, x_1 is the main factor of differentiation between the models. Furthermore, in our model the segmentation is made between three types of clients (loyal, poached and new), while in Gehrig *et al.* (2011) there are only two groups of clients (old and new). Lastly, Gehrig *et al.* (2011) does not consider the symmetric scenario. For comparison purposes, we will assume that those clients who, in our two-set ups, were, in the past, clients of firm B would, in Gehrig *et al.* (2011), be clients of the monopolist firm. In other words, even though they had a preference for firm B, they would not abandon the market as long as there was a firm

[11] Such setting can be applied to situations in which the management of a firm changes and inherits a certain market or to regulated markets facing a new incentive to the liberalization of competition.

producing the good. Therefore, p_b and q_a will be compared with p_{bg} and p_{ag}, respectively. p_{bg} and p_{ag} are the prices in Gehrig *et al.* (2011).

For simplification, we have considered similar costs for both firms. Therefore, in the following comparison, we will assume the marginal costs in Gehrig *et al.* (2011) to be equal as well.

As in the previous section, we will assume $\tau = 1$, $\sigma = 0.5$ and $c=0$ to draw practical conclusions from the expressions obtained while comparing the models.

3.4.1. Asymmetric behaviour-based pricing in both models

The equilibrium pricess in Gehrig *et al.* (2011) are:

$$p_{ag} = \tau + \frac{(2+\theta)\sigma+6c}{6}$$

$$z_{ag} = \tau + \frac{6c-(1-\theta)\sigma}{6}$$

$$p_{bg} = \tau + \frac{3c-(1-\theta)\sigma}{3}$$

Proof: see Appendix

The equilibrium profits in Gehrig *et al.* (2011) are:

$$\pi_{ag} = -\frac{1}{72}\frac{-4\sigma^2-36\tau^2+5\theta^2\sigma^2-24\sigma\tau-\theta\sigma^2+8c^2+24\theta\sigma\tau}{\tau}$$

$$\pi_{bg} = \frac{-((1-\theta)\sigma+3\tau)^2}{18\tau}$$

Proof: see Appendix

The consumer surplus in Gehrig *et al.* (2011) is:

$$CS_g = \beta + \frac{8c^2-8c(c+(1-\theta)\sigma+9\tau)-8c(9\tau-(1-\theta)\sigma)+\theta\sigma(\sigma+72\tau)+4(\sigma^2-18\sigma\tau-45\tau^2)-5\theta^2\sigma^2}{144\tau}$$

Proof: see Appendix

Comparing the prices from both models, we can observe that in our model (see proposition 1) all consumers pay a smaller price than in Gehrig *et al.* (2011).

Given the equilibrium profits found on section 3.1 and taking in consideration the assumptions about τ, σ and c, the profits of firm A are smaller than the profits in Gehrig *et al.* (2011)'s static model. For most values, the same remains true for the profits of firm

B. However, when firm A has a very strong base of loyal clients from the past (high x_1), especially when accompanied by a small percentage of new clients in the market, firm B can have higher profits in our model. In the literature when there is more than one period and, consequently, former customer decisions become a necessary part of the pricing strategy, firms are not able to maintain profits as high as those achieved in a single period.[12] Our model, although static, seems to corroborate this idea for the most part, as the consideration of past decisions as part of the pricing strategy appears to be detrimental for the firms in the great majority of situations. (as opposed to the situation in which those decisions are not considered). However, it does not assume it as a necessary result. In situations in which clients have formerly shown a clear preference for the dominant firm (a situation very close to a monopoly), the profits of its rival can be higher when past decisions are taken into consideration in the pricing strategies.

The opposite happens to the consumer surplus. When past decisions are considered in the model, customers are benefitted because they face lower prices in our asymmetric model.

The proof of this section's results can be found in Appendix.

[12] This result is in line, for example, with Fudenberg and Tirole (2000).

3.4.2. Symmetric behaviour-based pricing

We will now proceed to comparing Gehrig *et al.* (2011) with the version of our model presented in section 3.2.

From the comparison between the equilibrium prices in Gehrig *et al.* (2011) and those on proposition 2, it is possible to conclude that old consumers who buy from A always pay a lower price in our model. As for the clients of firm B, poached clients also pay a lower price in our model, but loyal clients may face a higher price (which happens when firm A has a relatively small base of loyal clients from the previous period). New consumers pay higher prices in our model, regardless of the firm they prefer.

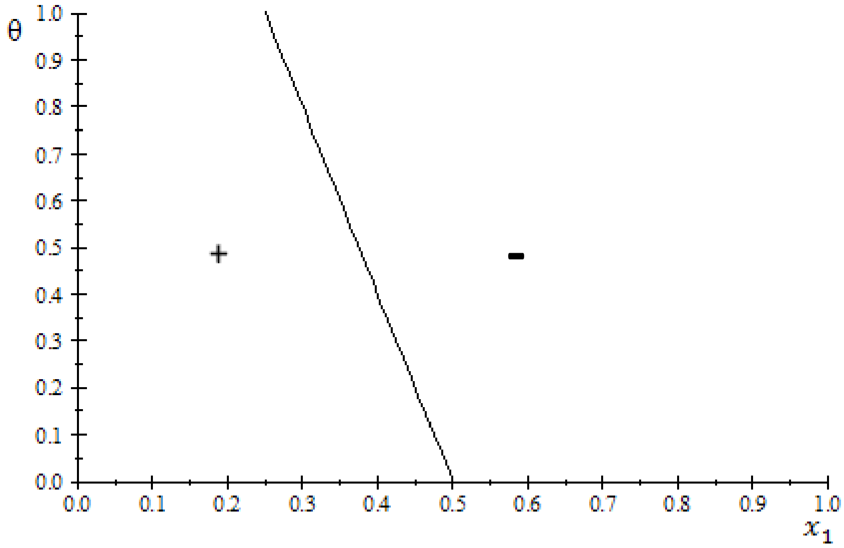

Figure 11 – Sign of the difference between the price paid by clients loyal to firm B in our behaviour-based pricing model and in Gehrig *et al.* (2011).

Considering the equilibrium profits found in both models, we cannot draw an immediate conclusion about which scenario is most benefits the firms. Assuming $\tau = 1$, $\sigma = 0.5$ and c=0, we have that the profits of the incumbent firm are almost always higher in Gehrig *et al.* (2011), which is understandable given the fact that the rival is unable to price discriminate in the model. However, for values of x_1 very close to one, the opposite happens. These are situations in which the base of loyal clients of firm A is so high that we are close to Gehrig et al. (2011)'s assumption of the previous existence of a monopoly. In fact, the big difference between the two modelling options is the consideration of an inherited loyal base of clients (x_1) towards firm A. Those loyal clients, who face switching costs, seem to be enough to guarantee higher profits for firm A, regardless of firm B's ability to price discriminate. In the same way, firm B can either benefit or not from the possibility of price discriminating. For most values of x_1 and θ, firm B is benefitted in our symmetric behaviour-based pricing model as opposed to the asymmetric model presented in Gehrig *et al.* (2011). However, when x_1 is relatively high but not overly high and θ is also not too high, the contrary can happen. In fact, there is an interval of values in which firm B is better off in Gehrig *et al.* (2011) than in the symmetric price discrimination set-up of our model. This means that if firm A has a relatively strong but not too strong previous loyal client base and the segment of new clients is not too big, firm B may be harmed by its own ability to price discriminate. This may be a result of including past decisions in the firms' pricing strategies in our model.

Consumers are always better off in our behaviour-based price discrimination model than in Gehrig *et al.* (2011).

The differences between the conclusions of the two models show both the importance of considering choices made in the past and the impact of eliminating an information asymmetry between firms (that may exist as a result, for example, of entering the market at different times or having access to different technologies).

The proof of the results presented in the section can be consulted in Appendix.

4. Conclusion

In this dissertation, we studied the effects of history-based price discrimination on consumers and firms in a context where firms had an inherited base of loyal clients. We also analyzed the impact of an information asymmetry regarding the clients' past behaviour. In fact, we presented two scenarios of a duopoly market: one where such asymmetry was present, with only one of the firms being capable of price discriminating, and another where both firms had an equal ability to use that pricing strategy. This can be interpreted as a comparison between a scenario where a relatively new firm in the market still has no access to client's history unlike the older firm (the information asymmetry is lasting) and a scenario where the asymmetry immediately disappears. Both these situations were also compared to a similar model, Gehrig *et al.* (2011), where former consumer decisions are not considered.

Regarding the asymmetric price discriminating model, we concluded that having a strong base of loyal clients is a strategic advantage for the price discriminating firm, as it creates conditions for stronger poaching strategies: an increase on the prices set to the loyal clients allows the setting of lower prices to the rival's clients. Substitution costs are an instrument that can be used to apply such strategies (an increase on these costs may result in lower prices to the clients the firm aims to poach, which is compensated by an increase on the prices to loyal clients). Also, unlike the standard Hotelling results, an increase on transportation costs does not necessarily result on an increase of prices. Indeed, when the price discriminating firm has a strong dominance, an increase on those costs may be fully supported by the loyal clients, allowing the firm to, once again, set lower prices to its rival's clients for poaching purposes.

However, it is when comparing this set-up with the set-up without an information asymmetry between firms that some of the most interesting conclusions emerge. For instance, it becomes clear that both the expansion of the market and the consumers' past decisions have important implications in the profitability of the firms. With an information asymmetry between the two firms, prices are higher with the entry of new clients. This entry always benefits the non-discriminating firm, but, in many cases, harms the price discriminating firm. The reason behind this is the fact that the entry of new customers in the market means a smaller percentage of old customers, which reduces the portion of loyal clients (and, consequently, the associated strategic advantage). Without any information asymmetry, conclusions are not very different. Even though there is no longer any difference in the ability of strategically using price discrimination for poaching reasons, the expansion of the market still influences the firms' performance. For instance, when one firm has an inherited position significantly stronger than the other, only the smaller firm benefits from the expansion of the market. This seems to indicate that the entry of new consumers in a market plays an important role to attenuate the dominance of a firm. Without it, that dominance might prevail over time and limit the expansion of smaller firms. Thus, in both the cases, the entry of new customers works a tool to reduce the advantage of the most powerful firms, whether that advantage comes from the access to different technologies or from having a much bigger market share.

Globally, the symmetric model always assures higher profits for the firm that, in the asymmetric model, was unable to price discriminate, which does not come as a surprise. For the firm that always has the ability of price discriminating, that is not always the case. In fact, both scenarios can happen. For its profits to be higher without the information asymmetry, it is necessary to have a strong base of loyal clients or a significant entry of new customers in the market. For the customers it seems preferable to have a scenario with information asymmetry, unless the market is almost stagnant and the dominant firm has a strong dominance (but not close to being a monopoly). This appears to have a direct connection with the behaviour of the prices. The prices for the new customers are always higher when both firms price discriminate, but for old customers that depends of the entry of new clients and the past decisions. When all prices for old customers are smaller in the symmetric model, customers are better-off with both firms price discriminating. However, as mentioned, this situation only happens under very specific conditions. So, it might be of the interest of consumers to keep their preferences (purchasing history) hidden as much as possible to, at least, make it difficult to newer firms to have access to it, especially in growing markets. New consumers, for example, have no interest of letting the firms know they have never participated in the market before. This might be accomplished, for example, through the possibility of deleting cookies. If consumers are capable of erasing the track of their previous purchases, it becomes much harder for firms to segment their clients and set different prices for each segment.

Different conclusions were found when comparing our symmetric model with Gehrig *et al.* (2011), which, once again, shows how considering past decisions changes the conclusions of the models. In fact, when using Gehrig *et al.* (2011) instead of our asymmetric set-up, we found eliminating the information asymmetry to always be beneficial for the consumers. As for the firms, the conclusions regarding the profits of the price discriminating firm were very similar. However, in some situations, the other firm was found to profit more when incapable of price discriminating, unlike what happened when considering our asymmetric model.

Lastly, through the comparison of our asymmetric model with Gehrig *et al.* (2011), it becomes clear that incorporating past decisions into the pricing strategies is beneficial for consumers, as it lowers all prices. Firms on the other hand are harmed by this factor, as predicted in the existent literature. Our model, however, finds an exception to these results. When there is a strong preference for the price discriminating firm, the other firm might benefit from considering past decisions when setting the prices.

In this dissertation, we have analyzed the impact of behaviour-based price discrimination in a context of information asymmetry. For that purpose, we used a static approach that assumed previous interaction on the market. Therefore, we considered an exogenous market-share (interpreted as the loyal base of clients of one of the firms). This leaves space for future research in this area, namely through a similar study in a dynamic setting to verify how our conclusions hold in a longer horizon.

5. Appendix

Proof of proposition 1

In game theory, the best response is the strategy or strategies that produce the most favourable outcome for a player, given the strategies of the other players. This is a key concept to find the Nash equilibrium, which is the solution in which all players are using the best possible strategy and, therefore, have nothing to gain by changing it, considering the opponent's decisions.

Through the maximization of the profit functions of both firms in order to the each of the prices set by them, we can find the best response functions. The best response functions in this model are:

$$\mathrm{p_a} = \frac{1}{2}c + \frac{1}{2}\sigma + \frac{1}{2}\tau + \frac{1}{2}\mathrm{p_b}; \quad \mathrm{q_a} = \frac{1}{2}c - \frac{1}{2}\sigma + \frac{1}{2}\tau + \frac{1}{2}\mathrm{p_b} - \tau x_1; \quad \mathrm{z_a} = \frac{1}{2}c + \frac{1}{2}\tau + \frac{1}{2}\mathrm{p_b}$$

$$\mathrm{p_b} = -\frac{1}{(2\theta-4)}(2c + \mathrm{p_a} + \mathrm{q_a} - c\theta + \theta\tau + 2\tau x_1 - \theta\mathrm{p_a} + \theta\mathrm{z_a} - 2\theta\tau x_1)$$

These functions show that the prices from firm A don't interact with each other. Instead, each of them interacts with $\mathrm{p_b}$ individually, as if we had different segments with an independent size. It is interesting to notice that in the market of firm B's old clients, firm A sets a lower price with the switching costs and sets a discount with x_1. This means that the higher x_1 is, the lower will $\mathrm{q_a}$ be. This might happen because when x_1 is high, it is expected that the rival's clients face a big transportation cost to buy from A. To oppose this effect, firm A sets a lower price for them. Firm B also sets a discount associated with x_1, in order to poach the maximum possible clients from firm A.

Also, the best response functions allow us to obtain proposition 1. We do so by solving a four-equation system with those functions.

Proof of the result in (8)

In order to determine the consumer surplus, we use the following expression:

Consumer Surplus = $(1 - \theta)\ (\int_0^{x_a}(\beta - p_a - \tau x)dx + \int_{x_a}^{x_1}(\beta - p_b - \tau(1-x) - \sigma)dx + \int_{x_1}^{x_b}(\beta - q_a - \tau x - \sigma)dx + \int_{x_b}^{1}(\beta - p_b - \tau(1-x))dx) + \theta(\int_0^{x_n}(\beta - z_a - \tau x)dx + \int_{x_n}^{1}(\beta - p_b - \tau(1-x))dx)$

Using the prices found in proposition 1 and the resulting expressions of the various indifferent consumers, we get (8).

Proof of proposition 2

To find the best response functions, we have to maximize the profits expression of each firm in order to each of the possible prices.

The best response functions in this model are:

$p_a = \frac{1}{2}c + \frac{1}{2}\sigma + \frac{1}{2}\tau + \frac{1}{2}q_b$;

$p_b = \frac{1}{2}c + \frac{1}{2}\sigma + \frac{1}{2}\tau + \frac{1}{2}q_a$;

$q_a = \frac{1}{2}c - \frac{1}{2}\sigma + \frac{1}{2}\tau(2x_1 - 1) + \frac{1}{2}p_a$;

$q_b = \frac{1}{2}c - \frac{1}{2}\sigma - \frac{1}{2}\tau(2x_1 - 1) + \frac{1}{2}p_b$

$z = \frac{1}{2}c + \frac{1}{2}\tau + \frac{1}{2}z_b$;

$z_b = \frac{1}{2}c + \frac{1}{2}\tau + \frac{1}{2}z_a$

These functions show the prices only interact in groups of two, as if we had different segments with an independent size. It is interesting to notice that in the market of firm A's old clients, firm B sets a lower price with the switching costs and sets a discount if $x_1 < 0.5$. Otherwise, the price increases with x_1. The opposite happens with price set by firm A to the old clients of firm B. A possible explanation for this fact might be that when a firm has a big base of loyal clients the main concern will be to keep those clients instead of poaching new ones to avoid an unnecessary price war. When the loyal clients are only a small portion of the total of clients in the market, there will be a need to practice lower prices to extend the number of clients of the firm.

By solving a system with the best response functions, we can obtain proposition 2.

Proof of the result in (14)

In order to determine the consumer surplus, we use the following expression:

Consumer Surplus = $(1-\theta)\left(\int_0^{x_a}(\beta - p_a - \tau x)dx + \int_{x_a}^{x_1}(\beta - q_b - \tau(1-x) - \sigma)dx + \int_{x_1}^{x_b}(\beta - q_a - \tau x - \sigma)dx + \int_{x_b}^{1}(\beta - p_b - \tau(1-x))dx\right) + \theta(\int_0^{x_n}(\beta - z_a - \tau x)dx + \int_{x_n}^{1}(\beta - z_b - \tau(1-x))dx)$

Using the prices found in proposition 2 and the resulting expressions of the various indifferent consumers, we get (14).

Proof of section in 3.3

We start by calculating the differences between the prices practiced in the symmetric behaviour-based price discrimination model (indentified with a 2) and in the asymmetric behaviour-based price discrimination model (identified with a 1):

$p_{a2} - p_{a1} = c + \frac{1}{3}\sigma + \frac{1}{3}\tau + \frac{2}{3}\tau x_1 - [c + \frac{\sigma}{2} + \frac{1}{3}\tau\frac{\theta - x_1 + \theta x_1 - 4}{\theta - 2}] = -\frac{1}{6}\frac{-2\sigma - 4\tau + \theta\sigma + 6\tau x_1 - 2\theta\tau x_1}{\theta - 2}$

$q_{a2} - q_{a1} = c - \frac{1}{3}\sigma + \tau - \frac{4}{3}\tau x_1 - [c - \frac{\sigma}{2} - \frac{1}{3}\tau\frac{-\theta - 5x_1 + 2\theta x_1 + 4}{\theta - 2}] = \frac{1}{6}\frac{-2\sigma - 4\tau + \theta\sigma + 4\theta\tau + 6\tau x_1 - 4\theta\tau x_1}{\theta - 2}$

$z_{a2} - z_{a1} = c + \tau - [c + \frac{1}{3}\tau\frac{\theta - x_1 + \theta x_1 - 4}{\theta - 2}] = -\frac{1}{3}\tau(x_1 - 2)\frac{\theta - 1}{\theta - 2}$

$p_{b2} - p_{b1} = c + \frac{1}{3}\sigma + \tau - \frac{2}{3}\tau x_1 - [c + \frac{1}{3}\tau\frac{-\theta - 2x_1 + 2\theta x_1 - 2}{\theta - 2}] = \frac{1}{3}\frac{-2\sigma - 4\tau + \theta\sigma + 4\theta\tau + 6\tau x_1 - 4\theta\tau x_1}{\theta - 2}$

$q_{b2} - p_{b1} = c - \frac{1}{3}\sigma - \frac{1}{3}\tau + \frac{4}{3}\tau x_1 - [c + \frac{1}{3}\tau\frac{-\theta - 2x_1 + 2\theta x_1 - 2}{\theta - 2}] = -\frac{1}{3}\frac{-2\sigma - 4\tau + \theta\sigma + 6\tau x_1 - 2\theta\tau x_1}{\theta - 2}$

$z_{b2} - p_{b1} = c + \tau - [c + \frac{1}{3}\tau\frac{-\theta - 2x_1 + 2\theta x_1 - 2}{\theta - 2}] = -\frac{2}{3}\tau(x_1 - 2)\frac{\theta - 1}{\theta - 2}$

Making each expression equal to zero, assuming $\tau = 1$, $\sigma = 0.5$ and $c = 0$, and solving it in order to θ, we obtain the values of θ for which there is no difference between the given prices, as a function of x_1. Then, we can easily conclude for which intervals each difference is positive or negative. It is important to remember that both x_1 and θ belong to the interval]0 ,1[. The conclusions were the following:

$p_{a2} - p_{a1}$ depends on x_1 and θ

$q_{a2} - q_{a1}$ depends on x_1 and θ

$z_{a2} - z_{a1} > 0$

$p_{b2} - p_{b1}$ depends on x_1 and θ

$q_{b2} - p_{b1}$ depends on x_1 and θ

$z_{b2} - p_{b1} > 0$

For a better understanding of the signs of each expression, the expressions were plotted and we got figures 5 – 8.

We follow the same reasoning for both the profits and the consumer surplus:

$$\pi_{a2} - \pi_{a1} = \frac{1}{36}(\theta\text{-}1)\ (\frac{-10\sigma^2+8\tau^2-16\sigma\tau+54\tau^2x_1{}^2+5\theta\sigma^2-48\tau^2x_1+32\theta\tau^2x_1+8\theta\sigma\tau-32\theta\tau^2x_1{}^2}{\tau(\theta-2)}$$

$$+\frac{12\sigma\tau x_1-6\theta\sigma\tau x_1}{\tau(\theta-2)})$$

$$\pi_{b2} - \pi_{b1} = -\frac{1}{9}(\theta\text{-}1)\ (\frac{-2\sigma^2-8\tau^2-8\sigma\tau-18\tau^2x_1{}^2+\theta\sigma^2+24\tau^2x_1-8\theta\tau^2x_1+4\theta\sigma\tau+8\theta\tau^2x_1}{\tau(\theta-2)}$$

$$+\frac{12\sigma\tau x_1-6\theta\sigma\tau x_1}{\tau(\theta-2)})$$

$$CS_2 - CS_1 = \frac{1}{72}(\theta-1)\frac{-8\sigma^2-11\tau^2-8\sigma\tau-36\tau^2x_1{}^2+4\theta\sigma^2+36\tau^2x_1-14\theta\tau^2x_1+4\theta\sigma\tau+14\theta\tau^2x_1{}^2}{\tau(\theta-2)}$$

After equalizing each expression to zero:

$\pi_{a2} - \pi_{a1}$ depends on x_1 and θ

$\pi_{b2} - \pi_{b1} > 0$

$CS_2 - CS_1$ depends on x_1 and θ

For a better understanding of the signs of the expressions $\pi_{a2} - \pi_{a1}$ and $CS_2 - CS_1$ we plotted them and got figures 9 and 10.

Proof of 3.4

From Gehrig *et al.* (2011) we have:

$$p_{ag} = \tau + \frac{(2+\theta)\sigma+4c_a+2c_b}{6}$$

$$z_{ag} = \tau + \frac{4c_a+2c_b-(1-\theta)\sigma}{6}$$

$$p_{bg} = \tau + \frac{3c_b + c_a - (1-\theta)\sigma}{3}$$

$$\pi_{ag} = \frac{(4(c_a - c_b)^2 - 8c_a(c_b + (1-\theta)\sigma + 3\tau) + 8c_b((1-\theta)\sigma + 3\tau) + \theta\sigma(\sigma - 24\tau) + 4(\sigma^2 + 6\sigma\tau + 9\tau^2) - 5\theta^2\sigma^2)}{72\tau}$$

$$\pi_{bg} = \frac{(c_a - c_b - (1-\theta)\sigma + 3\tau)^2}{18\tau}$$

$$CS_g = \beta + \frac{4(c_a^2 + c_b^2) - 8c_a(c_b + (1-\theta)\sigma + 9\tau) - 8c_b(9\tau - (1-\theta)\sigma) + \theta\sigma(\sigma + 72\tau) + 4(\sigma^2 - 18\sigma\tau - 45\tau^2) - 5\theta^2\sigma^2}{144\tau}$$

In our model, we assume both firms face the same marginal production costs. In order to better compare these results with our results, we will make the same assumption for Gehrig *et al.* (2011). Under our assumption ($c_a = c_b = c$), we have:

$$p_a = \tau + \frac{(2+\theta)\sigma + 6c}{6}$$

$$z_a = \tau + \frac{6c - (1-\theta)\sigma}{6}$$

$$p_b = \tau + \frac{3c - (1-\theta)\sigma}{3}$$

$$\pi_a = -\frac{1}{72}\,\frac{-4\sigma^2 - 36\tau^2 + 5\theta^2\sigma^2 - 24\sigma\tau - \theta\sigma^2 + 8c^2 + 24\theta\sigma\tau}{\tau}$$

$$\pi_b = \frac{-((1-\theta)\sigma + 3\tau)^2}{18\tau}$$

$$CS = \beta + \frac{8c^2 - 8c(c + (1-\theta)\sigma + 9\tau) - 8c(9\tau - (1-\theta)\sigma) + \theta\sigma(\sigma + 72\tau) + 4(\sigma^2 - 18\sigma\tau - 45\tau^2) - 5\theta^2\sigma^2}{144\tau}$$

Proof of section in 3.4.1

We start by calculating the differences between the profits in our asymmetric behaviour-based price discrimination model (indentified with a 1) and in Gehrig *et al.* (2011)'s model (identified with a g). Then, we do the same for the consumer surplus:

$p_{a1} - p_{ag} = c + \frac{\sigma}{2} + \frac{1}{3}\tau\frac{\theta - x_1 + \theta x_1 - 4}{\theta - 2} - [\tau + \frac{(2+\theta)\sigma + 6c}{6}] = -\frac{1}{6}(\theta - 1)\frac{-2\sigma + 4\tau + \theta\sigma - 2\tau x_1}{\theta - 2}$

$q_{a1} - p_{ag} = c - \frac{\sigma}{2} - \frac{1}{3}\tau\frac{-\theta - 5x_1 + 2\theta x_1 + 4}{\theta - 2} - [\tau + \frac{(2+\theta)\sigma + 6c}{6}] = -\frac{1}{6(\theta-2)}(3\theta\sigma - 4\tau - 10\sigma + 4\theta\tau - 10\tau x_1 + \theta^2\sigma + 4\theta\tau x_1)$

$z_{a1} - z_{ag} = c + \frac{1}{3}\tau\frac{\theta - x_1 + \theta x_1 - 4}{\theta - 2} - [\tau + \frac{6c - (1-\theta)\sigma}{6}] = -\frac{1}{6}\frac{-4\sigma - 4\tau + 4\theta\tau - 10\tau x_1 + \theta^2\sigma + 4\theta\tau x_1}{\theta - 2}$

$p_{b1} - p_{bg} = c + \frac{1}{3}\tau\frac{-\theta - 2x_1 + 2\theta x_1 - 2}{\theta - 2} - [\tau + \frac{3c - (1-\theta)\sigma}{3}] = \frac{1}{3}\frac{\theta - 1}{\theta - 2}(2\sigma - 4\tau - \theta\sigma + 2\tau x_1)$

$\pi_{a1} - \pi_{ag} = -\frac{1}{72\tau(\theta-2)}(-8c^2\theta + 16c^2 - 5\theta^3\sigma^2 + 29\theta^2\sigma^2 + 36\theta^2\sigma\tau x_1 - 24\theta^2\sigma\tau + 16\theta^2\tau^2 x_1^2 - 16\theta^2\tau^2 x_1 + 4\theta^2\tau^2 - 52\theta\sigma^2 - 108\theta\sigma\tau x_1 + 72\theta\sigma\tau - 68\theta\tau^2 x_1^2 + 80\theta\tau^2 x_1 + 4\theta\tau^2 + 28\sigma^2 + 72\sigma\tau x_1 - 48\sigma\tau + 52\tau^2 x_1^2 - 64\tau^2 x_1 - 8\tau^2)$

$\pi_{b1} - \pi_{bg} = -\frac{1}{18\tau}\frac{\theta-1}{\theta-2}(\theta^2\sigma^2 - 3\theta\sigma^2 + 6\theta\sigma\tau + 4\theta\tau^2 x_1^2 + \theta\tau^2 + 2\sigma^2 - 12\sigma\tau - 4\tau^2 x_1^2 - 8\tau^2 x_1 + 14\tau^2)$

$CS_1 - CS_g = \frac{1}{144}\frac{\theta-1}{\theta-2}(5\theta^2\sigma^2 - 24\theta\sigma^2 + 108\theta\sigma\tau x_1 - 72\theta\sigma\tau - 16\theta\tau^2 x_1^2 + 16\theta\tau^2 x_1 - 4\theta\tau^2 + 28\sigma^2 - 216\sigma\tau x_1 + 144\sigma\tau + 52\tau^2 x_1^2 - 160\tau^2 x_1 + 184\tau^2)$

Making each expression equal to zero, assuming $\tau = 1$, $\sigma = 0.5$ and $c = 0$ and solving it in order to θ, we obtain the values of θ for which the variables in both models share the same value, as function of x_1. From there, we can conclude for which intervals each difference is positive or negative. It is important to remember that both x_1 and θ belong to the interval]0 ,1[. The conclusions were the following:

$p_{a1} - p_{ag} < 0$

$q_{a1} - p_{ag} < 0$

$z_{a1} - z_{ag} < 0$

$p_{b1} - p_{bg} < 0$

$\pi_{a1} - \pi_{ag} < 0$

$\pi_{b1} - \pi_{bg}$ depends on x_1 and θ (we tested different values to get to the conclusions in our paper)

$CS_1 - CS_g > 0$

Proof of section in 3.4.2

The first step is to calculate the differences between the prices in our symmetric behaviour-based price discrimination model (indentified with a 2) and in Gehrig *et al.* (2011)'s model (identified with a g). Then, we do the same for the profits and the consumer surplus:

$$p_{a2} - p_{ag} = c + \frac{1}{3}\sigma + \frac{1}{3}\tau + \frac{2}{3}\tau x_1 - [\tau + \frac{(2+\theta)\sigma+6c}{6}] = \frac{1}{3}\sigma - \frac{2}{3}\tau + \frac{2}{3}\tau x_1 - \frac{1}{6}\sigma(\theta + 2)$$

$$q_{a2} - p_{ag} = c - \frac{1}{3}\sigma + \tau - \frac{4}{3}\tau x_1 - [\tau + \frac{(2+\theta)\sigma+6c}{6}] = -\frac{1}{3}\sigma - \frac{4}{3}\tau x_1 - \frac{1}{6}\sigma(\theta + 2)$$

$$z_{a2} - z_{ag} = c + \tau - [\tau + \frac{6c-(1-\theta)\sigma}{6}] = -\frac{1}{6}\sigma(\theta - 1)$$

$$p_{b2} - p_{bg} = c + \frac{1}{3}\sigma + \tau - \frac{2}{3}\tau x_1 - [\tau + \frac{3c-(1-\theta)\sigma}{3}] = \frac{1}{3}\sigma - \frac{2}{3}\tau x_1 - \frac{1}{3}\sigma(\theta - 1)$$

$$q_{b2} - p_{bg} = c - \frac{1}{3}\sigma - \frac{1}{3}\tau + \frac{4}{3}\tau x_1 - [\tau + \frac{3c-(1-\theta)\sigma}{3}] = \frac{4}{3}\tau x_1 - \frac{4}{3}\tau - \frac{1}{3}\sigma - \frac{1}{3}\sigma(\theta + 2)$$

$$z_{b2} - p_{bg} = c + \tau - [\tau + \frac{3c-(1-\theta)\sigma}{3}] = -\frac{1}{3}\sigma(\theta - 1)$$

π_{a2} - π_{ag} = $\frac{1}{72\tau}$ ($8c^2 + 5\theta^2\sigma^2 - 9\theta\sigma^2 - 48\theta\sigma\tau x_1 + 40\theta\sigma\tau - 80\theta\tau^2 x_1^2 + 80\theta\tau^2 x_1 - 4\theta\tau^2 + 4\sigma^2 + 48\sigma\tau x_1 - 40\sigma\tau + 80\tau^2 x_1^2 - 80\tau^2 x_1 + 4\tau^2$)

$\pi_{b2} - \pi_{bg} = -\frac{1}{18\tau}(\theta-1)(\sigma^2 + \tau^2 + 14\sigma\tau + 20\tau^2 x_1^2 + \theta\sigma^2 - 20\tau^2 x_1 - 12\sigma\tau x_1)$

$CS_2 - CS_g = -\frac{1}{144\tau}(\theta-1)(4\sigma^2 + 4\tau^2 + 8\sigma\tau - 208\tau^2 x_1^2 - 5\theta\sigma^2 + 208\tau^2 x_1)$

Making each expression equal to zero, assuming $\tau = 1$, $\sigma = 0.5$ and $c = 0$ and solving it in order to θ, we obtain the values of θ for which there is no difference between the prices, as function of x_1. Then, we can easily conclude for which intervals each difference is positive or negative. It is important to remember that both x_1 and θ belong to the interval]0 ,1[. The conclusions were the following:

$p_{a2} - p_{ag} < 0$

$q_{a2} - p_{ag} < 0$

$z_{a2} - z_{ag} > 0$

$p_{b2} - p_{bg}$ depends on x_1 and θ (we tested different values to get to the conclusions in our paper).

$q_{b2} - p_{bg} < 0$

$z_{b2} - p_{bg} > 0$

π_{a2} - π_{ag} depends on x_1 and θ (we tested different values to get to the conclusions in our paper).

$\pi_{b2} - \pi_{bg}$ depends on x_1 and θ (we tested different values to get to the conclusions in our paper).

$CS_2 - CS_g > 0$

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